Edmund John Senkler

Trin. Coll.

Cambridge

CAMBRIDGE

PROBLEMS.

1801—1820.

UNIVERSITY OF CAMBRIDGE.

1820.

A LIST of the MODERATORS from the year 1800 to the present time: with a Summary of the ACADEMICAL HONOURS obtained during that period.

	Wranglers.	Sen^{r} Opt.	Jun^{r} Opt.	Total.
1801.				
John Walker, M. A. Pet. Coll. T. W. Hornbuckle, M. A. Joh.	17	17	9	43
1802.				
W. Dealtry, M. A. Trin. Coll. John Palmer, M. A. Joh.	16	16	12	44
1803.				
Robert Woodhouse, M. A. Caius T. W. Hornbuckle, M. A. Joh.	13	17	9	39
1804.				
John Hudson, M. A. Trin. Coll. Robert Woodhouse, M. A. Caius	16	15	8	39
1805.				
Thomas Sowerby, M. A. Queen's T. W. Hornbuckle, M. A. Joh.	15	10	8	33
1806.				
G. Barnes, M. A. Queen's Coll. John Brown, M. A. Trin. Coll.	14	14	7	35
1807.				
Robert Woodhouse, M. A. Caius G. D'Oyly, M. A. Bene't Coll.	12	14	8	34
1808.				
Robert Woodhouse, M. A. Caius G. D'Oyly, M. A. Bene't Coll.	17	16	5	38
1809.				
John Brown, M. A. Trin. Coll. G. D'Oyly, M. A. Bene't Coll.	13	13	10	36

	Wrang-lers.	Sen^{r} Opt.	Jun^{r} Opt.	Total.
1810.				
T. Turton, M. A. Cath. Hall. Henry Walter, M. A. Joh.	17	13	15	45
1811.				
Thomas Jephson, M. A. Joh. T. Turton, M. A. Cath. Hall.	15	17	11	43
1812.				
T. Turton, M. A. Cath. Hall. J. D. Hustler, M. A. Trin. Coll.	17	18	8	43
1813.				
Thomas Jephson, M. A. Joh. G. Macfarlan, M. A. Trin. Coll.	15	14	15	44
1814.				
G. Macfarlan, M. A. Trin. Coll. Miles Bland, M. A. Joh.	19	17	13	49
1815.				
Miles Bland, M. A. Joh. William Hustler, M. A. Jesus	22	23	9	54
1816.				
Miles Bland, M. A. Joh. William French, M. A. Pemb.	19	15	14	48
1817.				
John White, M. A. Caius G. Peacock, M. A. Trin. Coll.	18	18	12	48
1818.				
Fearon Fallows, M. A. Joh. William French, M. A. Pemb.	28	30	11	69
1819.				
Richard Gwatkin, M. A. Joh. G. Peacock, M. A. Trin. Coll.	19	23	14	56
1820.				
Henry Wilkinson, M. A. Joh. W. Whewell, M. A. Trin. Coll.	18	19	15	52

CAMBRIDGE
Problems.

1801

Morning Problems.—Mr. HORNBUCKLE.

FIRST AND SECOND CLASSES.

1. COMPARE the velocities acquired in falling freely from different altitudes towards *different* centers of force, the law of force being the inverse square of the distance.

2. Investigate the equation to the reciprocal spiral, and thence determine the law of the force by which a body may describe the curve.

3. Let a given parabola be just immersed vertically in a fluid; at what distance from the vertex must a line be drawn parallel to its base, that the pressure on the upper part may be to that on the lower in the proportion of m to n?

4. A body falls freely by the force of gravity

down AB, and uniformly describes the space BD, equal to twice AB, on the horizontal plane, with the velocity acquired. Determine, geometrically, the length and inclination of a plane drawn from A to BD, the time down which may be equal to the time down AB and along BD.

5. Divide a given angle into two angles, whose tangents shall be to each other in a given ratio.

6. On a given day, in a given latitude, the length of the shadow cast by a tower at 12 o'clock was (a) feet. Required the height of the tower.

7. Find the length of an arc of the meridian corresponding to any given latitude, according to Mercator's projection; and reconcile it to the construction given by Cotes.

8. Where must an eye be placed that an object, seen through a double convex lens, may appear of the same magnitude at all distances from the lens?

9. A body is projected from the top of a given inclined plane. Required the direction of projection in which the least velocity will bring it to the bottom of the plane. Required also this velocity.

10. Compare the values of the respective infinite series $1+2x+3x^2+4x^3+\&c.$ and $1-2x+3x^2-4x^3+\&c.$

11. Compare the chances of throwing an ace in two trials with one die, and in one trial with two.

12. Transform the cubic $x^3-px^2+qx-r=0$, whose roots are a, b, c, into one whose roots are $\frac{1}{a+b}$, $\frac{1}{a+c}$, $\frac{1}{b+c}$.

13. Any two sides of a spherical triangle remaining constant, determine the ratio of the nascent

increments of the angle included between those sides, and of either of the other angles.

14. Suppose a given sphere to be projected in a medium whose density is $\frac{1}{3}$ that of itself; compare the velocity of projection with that remaining in the sphere after describing a space equal to eight diameters; the resistance to the motion arising only from the inertia of the medium.

Third and Fourth Classes.—Mr. Hornbuckle.

1. Required the price of paving a floor, whose length is 10yds. 2ft. and breadth 5yds. 1ft. at 2*s.* per square foot.

2. Reduce 17*s.* $9\frac{1}{2}$*d.* to the decimal of a pound.

3. Prove the rule for finding the greatest common measure of two quantities; and reduce $\frac{216}{3186}$ to its lowest terms.

4. If an angle of a triangle be bisected by a line which also cuts the base, the rectangle under the sides of the triangle is equal to the rectangle under the segments of the base, augmented by the square of the bisecting line.

5. Given the three angles of a triangle and the radius of the circumscribing circle, to find the sides.

6. The sum of the tangents of two arcs : diff. of tangents :: the sine of the sum of those arcs : sine of the difference.

7. Prove that a body cannot easily be balanced on a point under the center of gravity.

8. Given the length of a pendulum (l) that os-

cillates seconds, find by means of it how far a body will fall freely by the force of gravity in t''.

9. Given the velocity and direction of projection, find the range on a plane of given elevation, and the greatest altitude.

10. Find the fluxions of the following quantities $\frac{x}{(a^2-x^2)^{\frac{1}{2}}}$, $\frac{\sqrt{a^2+x^2}}{\sqrt{a^2-x^2}}$, a^x, a being constant.

11. Inscribe the greatest cone in a given sphere.

12. Find the chords of curvature perpendicular and parallel to the axis at a given point of the common parabola.

13. The roots of the cubic $x^3-px^2+qx-r=0$ are in musical progression, shew how they may be found.

14. Find generally the sums of the powers of the roots of an equation.

15. Explain the principle of the hydrometer, and shew that if it be made to sink to the same depth in different fluids, the specific gravities of these fluids are as the weights of the instrument in the several cases.

16. Find the center of a Meniscus, and prove it to be a fixed point.

17. Place an object before a double convex lens, so that the image may be double of the object, and erect.

18. Given the right ascension and declination of a star. Required its latitude and longitude.

19. Supposing the earth's orbit a parabola, find the apparent path of aberration of a fixed star parallel to the ecliptic.

20. The velocity at any point of a conic section : velocity in a circle at the same distance :: $\sqrt{\frac{1}{2}L \times SP} : SY$; Prove this, and shew it to be the same ratio as that of $\sqrt{HP} : \sqrt{AC}$, AC being the semi-axis major.

21. How far must a body fall internally to acquire the velocity in a parabola, the force varying inversely as the square of the distance?

22. Compare, by means of a circle, the times of describing freely different spaces from the same distance towards the same center of force, the law of the force being the inverse square of the distance.

23. Explain the principle of Cassegrain's telescope, and find its magnifying power.

24. Sum the following series $1.2.4 + 2.3.5 + 3.4.6 + \&c.$ to n terms.

Also $\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \frac{1}{4.6} + \&c.$ *ad inf.*

Fifth and Sixth Classes.—Mr. HORNBUCKLE.

1. $17\frac{1}{2}$ ells of cloth, each containing 5qrs. cost £.6. 17. $10\frac{3}{4}$; how much will 18 yards cost at the same rate?

2. Required the interest due on £.115. for $5\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. per ann. simple interest.

3. Solve the following equations,

$$\sqrt{x} - 4 = \frac{259 - 10x}{4 + \sqrt{x}}.$$

$$\frac{10}{x} - \frac{14 - 2x}{x^2} = 2\frac{4}{9}.$$

4. In any plane oblique-angled triangle, given two sides and the included angle, to solve the triangle.

5. A body falls from rest by the force of gravity down a given inclined plane; compare the times of describing the first and last halves of it.

6. Prove the velocity in any point of a parabola equal to that acquired in falling down $\frac{1}{4}$ the parameter.

7. Given a rectilinear object, and its inclination to a known refracting surface; required the magnitude and inclination of the image.

8. Prove that the image of a straight line before a spherical reflector is the arc of a conic section.

9. Find the time of emptying a cylinder of given base and altitude through a small orifice in its base; and investigate the fluxional expression for the time of emptying vessels in general.

10. If all the coefficients of an equation be whole numbers, shew that it cannot have a fractional root.

11. Draw a meridian line on a horizontal plane.

12. Compare the velocity in a curve with that in a circle, at the same distance, in general, and in the conic sections; the center of force being in the focus.

Evening Problems.—Mr. Walker.

1. Find the exact value of .1666, &c. of a £.

2. Find the length of a pendulum that would oscillate seconds at the dist. of the earth's rad. above its surface; and then determine the point below the surface where it would oscillate in the same time.

3. A body, projected from the top of a tower at an angle of 45° above the horizontal direction, fell in 5″ at a dist. from the bottom of the tower equal to its altitude: find that alt. in feet.

4. Compare the resistance upon the curve of a semi-circle, moving in a fluid in the direction of its axis, with the resistance upon its base.

5. Two equal hollow cubes are immersed in water, having a side parallel to the surface, and their depths are in the ratio of 4 : 1; compare the times of filling through equal orifices in the bottoms.

6. What must be the form of a glass lens placed in water, that all rays, incident parallel to its axis, may converge accurately to one point within the water?

7. The reflecting curve is a semi-circle, and rays fall parallel to its axis: construct the caustic; and compare the density of the rays at different points.

8. Given the Sun's alt. at 6 o'clock, and the alt. when due east; find the lat. of the place.

9. Given the *mean* horary motion of a planet in its orbit; shew the method of finding the horary motion in longitude.

10. The hyp. logarithm being given, find the corresponding number.

11. Sum the series

$$\frac{2}{1.3} \times \frac{1}{3} + \frac{3}{3.5} \times \frac{1}{3^2} + \frac{4}{5.7} \times \frac{1}{3^3} + \&c. \textit{ ad inf.}$$

and, $1.2.4 + 3.4.6 + 5.6.8 + 7.8.10 + \&c.$ to n terms, by increments.

12. Shew the method of finding the content of a pyramid, whatever be the figure of its base; the area of the base, and alt. of the solid, being given.

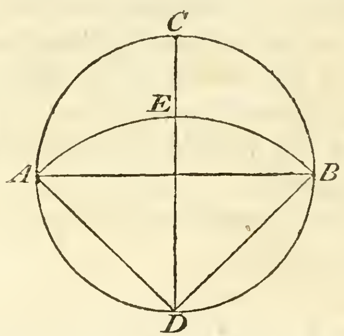
13. Investigate the ratio of the centripetal and centrifugal forces in any curve; and apply it to the hyperbolic spiral.

14. Find the actual velocity and periodic time of a body revolving in a circle at the dist. of the earth's rad. above its surface.

15. The equation to a certain curve is $y = \frac{4x^2}{\sqrt{a^2 - x^2}}$, where x is the abscissa and y the ordinate; find the whole area of the curve, and shew that it = area of a circle whose rad. is a.

16. Shew the method of comparing the mean addititious force of the sun upon the moon, with the force of gravity at the earth's surface.

17. Let AB and DC be two diameters of a given circle, drawn at right angles to each other; AEB a circular arc described with rad. DB or DA; prove



that the area of the lune $AEBC$ = area of triangle ADB.

18. Suppose a comet, in its descent towards the sun, to impel the earth from a circular orbit, in a direction making any acute angle with the earth's

distance from the sun; and the velocity after impact : velocity before :: $\sqrt{3} : \sqrt{2}$; find what change would be produced in the length of the year.

19. A globe and its circumscribed cylinder revolve uniformly round a common axis in the same time; compare the motion of the cylinder with that of the globe.

20. Shew that the fluent of $\frac{\dot{x}}{x}\int\frac{\dot{x}}{1+x}$, generated whilst x from 0 becomes 1, is $=\frac{Q^2}{3}$, where Q is the quadrantal arc of a circle whose radius is 1.

21. A body descends in a parabola in a resisting medium, by a force tending to the focus, which $\propto\frac{1}{\text{dist.}}$. Compare the resistance with the centripetal force; and find the law of the velocity.

22. The relation between the abscissa and ordinate of an algebraic curve is expressed by the equation $y^n-(a+bx).y^{n-1}+(c+dx+ex^2).y^{n-2}-\&c.=0$; prove that the sum of the ordinates divided by the respective subtangents, is a constant quantity.

23. A cylinder of a given weight and dimensions, is put in motion round an axis parallel to the horizon, by a given weight (P) suspended by a small string wound round the surface of the cylinder; find the actual time in which P would descend from the surface of the earth to the centre.

24. Let A and B be two particles of matter, con-

nected by an inflexible line AB; and let a force be impressed perpendicular to AB at any point D which is not the centre of gravity: find the point of initial spontaneous rotation; and then determine the path of that point in one revolution of A and B.

Morning Problems.—Mr. WALKER

FIRST AND SECOND CLASSES.

1. An equiangular prism is placed upon an inclined plane with its axis parallel to the horizon, and is just supported:—find the plane's inclination.

2. A cylinder is just immersed in a fluid with its axis perpendicular to the surface; find at what point it must be cut by a plane perpendicular to its axis, that the pressures upon the convex surfaces may be equal.

3. Prove that a small rectilinear object and image subtend equal angles at the vertex of a spherical reflector.

4. The latitude being given, find at what time on the longest day, the variation of the sun's altitude is greatest.

5. Given the moon's horizontal parallax and periodic time, with the sun's apparent $\frac{1}{2}$ diameter and length of a sidereal year; shew the method of comparing the densities of the earth and sun.

6. $a\dot{z} = \dot{y}\sqrt{a^2 + 4y^2}$, where z is the arc of a certain curve, and y the ordinate; determine the relation between the ordinate and abscissa.

7. Prove that the solid generated by the revolution of an ellipse round the minor axis, is a mean

proportional between the solid generated round the major axis and its circumscribed sphere.

8. Find the distance of the point of suspension from the centre of gravity of a given system of bodies, that the time of an oscillation may be the least possible.

9. Investigate and construct Cotes's fifth spiral.

10. The moon performs a revolution round the earth in a certain period, whilst they revolve round their common centre of gravity; in what ratio must the mean distance of the moon be diminished that it may revolve round the earth at *rest*, in the same time?

11. Describe the curve which is the locus of the equation $x^4 - a^2x^2 + a^2y^2 = 0$: and find its whole area.

12. Find the fluent of $\frac{\dot{x}}{(1+x^2)^2}$, having given the fluent of $\frac{\dot{x}}{1+x^2}$: find also the fluent of $v^2 x^3 \dot{x}$, where v = hyp. log. x.

13. Prove the differential method of summing series, by the method of increments. Emerson's Incr. p.

Or rather Colenso's Part II. p

14. Find the attraction of a corpuscle to a sphere, when the attractive force to each particle $\propto \frac{1}{\overline{\text{dist.}}|^3}$.

Third and fourth Classes.—Mr. Walker.

1. Find the interest of £555. for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ per cent.

2. A piece of ground, containing 4970.25 square

yards, is to be laid out in the form of a square: find the length of a side.

3. Given $x + \sqrt{a^2 + x^2} = \dfrac{2a^2}{\sqrt{x^2 + a^2}}$: find x.

4. Prove that the ratio $\sqrt[m]{1 + x} : 1$ is nearly equal to the ratio $1 + \dfrac{x}{m} : 1$, where x is very small when compared with 1.

5. Shew the method of ascertaining the height of a mountain above the level of an horizontal plane contiguous to a sloping side.

6. Prove that the minor axis of an ellipse is a mean proportional between the major axis, and the latus rectum.

7. In a right-angled spherical triangle, sine of hypothenuse : radius :: sine of a side : sine of the angle opposite that side. Required a demonstration.

8. Given the two apparent weights in each end of a pair of false scales, find the true weight; and shew whether it is greater or less than $\frac{1}{2}$ their sum.

9. Find the height to which a body will rise, if projected perpendicularly from the horizon, with a velocity of 144 feet in 1″: and find how far it will ascend in 2″.

10. Compare the time of an oscillation in a cycloid, with the time in which a body would fall through a space = length of the pendulum.

11. A cubical vessel, whose altitude is 32 inches, stands upon an horizontal plane and is filled with water:—find where a small orifice must be made in a side, that the fluid may spout to a distance equal to the height of the vessel.

12. Find the height of an homogeneous atmosphere; and shew that it is about $5\frac{1}{2}$ miles.

13. Find the principal focus of rays refracted through a sphere, denser than the ambient medium; and, supposing the focus to be in the surface of emergence, determine the ratio between the sines of incidence and refraction.

14. Determine the place of a double convex lens, between the eye of a spectator and an object at a given distance, that the apparent magnitude may be a maximum.

15. Construct two angles *geometrically*, whose sines are in the ratio $a : b$, and tangents in the ratio $m : 1$; and prove that rad. : cos. of greater angle $:: \sqrt{(m^2-1)b^2} : \sqrt{a^2-b^2}$.

16. Prove that $\dot{z} = \frac{r^2 \dot{s}}{s\sqrt{s^2 - r^2}}$, where r and s are radius and secant of the arc z.

17. Find the fluent of $\frac{z^{\frac{1}{2}n-1}\dot{z}}{\sqrt{a^n + z^n}}$; and of $v^n \dot{x}$ where $v =$ hyp. log. x.

18. Sum the series $\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} +$ &c. ad infinitum. And $1^2 + 2^2 + 3^2 +$ &c .... to n terms.

19. Construct an horizontal dial for a given latitude; and find the angle between the hour lines of 12 and 3.

20. Given the sun's declination and latitude of the place; find the time of its rising.

21. Shew that every equation, whose roots are possible, has as many changes of the signs as it has positive roots.

22. Prove that the periodic times in all ellipses round the same centre are equal; and, round different centres, that they $\propto \frac{1}{\sqrt{\text{abs. force}}}$.

23. Investigate the ratio of the angular velocities of the distance and perpendicular upon the tangent, in any curve; and apply it to the logarithmic spiral.

24. A body descends from a given altitude by a force which $\propto \frac{1}{\overline{\text{dist.}}|^2}$; and, at the middle point of its descent, is projected with the velocity acquired, in a direction making an acute angle with the distance from the centre:—what orbit will the body describe? and what will be its periodic time, when compared with the periodic time in a circle whose radius is the given altitude?

Evening Problems.—Mr. Hornbuckle.

1. Investigate the rule for extracting the cube root, and apply it to find the cube root of 738,763264.

2. Required the discount on a given sum (£p) due $1\frac{1}{2}$ years hence, at 5 per. cent. per ann.

3. Resolve the recurring series $a + bx + cx^2 + dx^3 + \&c.$ whose scale of relation is $f+g$, into two geometrical series.

4. Two equal weights are suspended by a string passing over three tacks, which form an isosceles triangle, the base being parallel to the horizon, and the vertical angle 120°. Compare the respective pressures on the tacks with each other, and with the weights.

5. Suppose a straight rod to be partly immersed in a vessel of water; determine the angle at which it must be inclined to the surface, that the apparent bending at the surface may be a maximum.

6. A cylinder full of water, whose length is equal to the diameter of the base, is supported with its sides parallel to the horizon: compare the time of discharging half the fluid through a small orifice in the lower side in this situation, with that of discharging the same quantity through an equal orifice in the base, when the sides are perpendicular to the horizon.

7. Apply Napier's rule to find the declination of a star, which, in a given latitude, rises in the north east point.

8. Investigate the nature of the curve in which a body descends from one given point to another in the least time possible; the velocity at each point being supposed to vary as the corresponding ordinate of the curve.

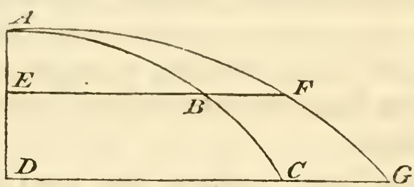
9. A cylindrical rod suspended at one end, whose weight is (W) and length (l) inches, oscillates seconds; on what part of the rod must a given weight (w) be suspended that it may oscillate twice in a second.

10. Approximate to the roots of the equation $x^2+xy=5$, $2xy-y^2=2$, and shew on what the accuracy of an approximation depends.

11. Find the area of the curve, whose equation is $y=a\times$ hyp. log. $\frac{a-\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}$, contained between

the values (a) and (b) of the abscissa; x being the abscissa and y the ordinate.

12. ABC is a semi-cycloid, DC its base, AD its axis, AFG a curve traced out by assuming the ordinate EF always equal to the cycloidal arc AB: investigate the equation to the curve AFG, and compare its area with the area of the cycloid.



13. Sum the following series $1.3^2 + 3.5^2 + 5.7^2 +$ &c. to n terms.

$$\frac{1}{1.2} - \frac{1}{2.4} + \frac{1}{3.6} - \frac{1}{4.8} + \text{\&c. ad inf.}$$

$$\frac{1}{1.2.4} - \frac{1}{2.3.5} + \frac{1}{3.4.6} - \text{\&c. ad inf.}$$

14. The roots of the equation $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$ are in arithmetical progression; the least root is $\frac{p}{n} - \frac{n-1}{n} . \sqrt{\frac{(n-1).3p^2 - 6nq}{n^2-1}}$, and the common difference $\frac{2}{n} . \sqrt{\frac{(n-1).3p^2 - 6nq}{n^2-1}}$, The investigation is required.

15. Find the fluents of the following quantities,

$$\frac{\dot{x}}{x^3 \times (a+x)^2} . \frac{b\dot{y}}{(1-y)^2 . \sqrt{1+y^2}} . \frac{a\dot{x}}{x\sqrt{a^2 - ax + x^2}} .$$

16. Prove that, when the first point of Aries rises, the ecliptic makes the least angle with the horizon,

and when it sets, the greatest; and thence explain the phænomenon of the harvest moon.

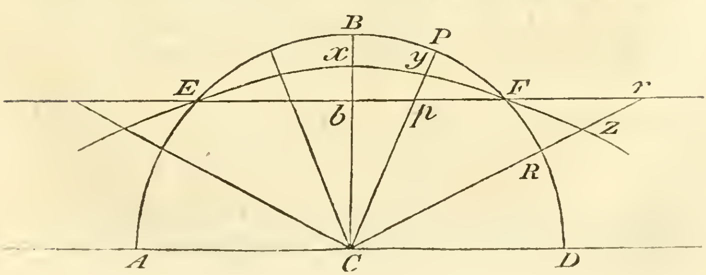
17. Suppose a body describing a logarithmic spiral in a resisting medium, the density varying inversely as the distance, and the centripetal force as the square of the density, to be deprived of its angular motion at a given distance from the centre; compare the time of its descent to the centre in a straight line, with the time of descent in the spiral.

18. Two bodies, whose weights are A and B, are projected together with the respective velocities, a and b, from the same point, in the same direction, and at a given angle of inclination to the horizon. Required the greatest altitude to which their common centre of gravity will ascend, and the path described by it.

19. A cylindrical vessel, of given altitude and base, is situated on a horizontal plane; an eye is placed so as to see only the farther extremity of that diameter of the base which passes through the point, in which a perpendicular drawn from the eye to the plane meets it. To what depth must it be filled with water, that the eye in the same situation may see the centre of the base?

20. Construct a vertical south-east dial.

21. ABD is a semi-circle, AD its diameter, EF



any chord parallel to AD, produced indefinitely, CB, CP, CR radii; the parts Bb, Pp, Rr of the radii, intercepted between the chord, or the chord produced, and the circumference, are bisected in x, y, z; find the nature of the curve passing through all the points of bisection, both within, and without the circle.

22. Investigate the number expressing the probability of throwing an ace at least (t) times in (n) trials with a die of (p) faces marked $1, 2, 3 \ldots p$.

23. Shew that Newton has properly applied the principles of the golden rule in his investigation of the ratio between the equatorial and polar diameters of the earth; i. e. if this ratio be that of $1+n : 1$, n being very small, and the figure of the earth an oblate spheroid, prove that the excess of weight supported at the equator is proportional to the difference (n) of the diameters.

24. Find at what angle a plane, which is perpendicular to the plane of the meridian, must be inclined to the horizon of a given place, that the diurnal path of the shadow of the extremity of an object, erected perpendicular to the horizon; may be a parabola on the plane: the sun's declination being given, and supposed invariable during the course of one day.

1802.

Morning Problems.—Mr. Palmer.

FIRST AND SECOND CLASSES.

1. Given the three angles of a plane triangle, and the radius of its inscribed circle, to determine its sides.

2. The specific gravities of two fluids, which will not mix, are to each other as $n : 1$, compare the quantities which must be poured into a cylindrical tube, whose length is (a) inches, that the pressures on the concave surfaces of the tube, which are in contact with the fluids, may be equal.

3. Determine that point in the arc of a quadrant from which two lines being drawn, one to the centre and the other bisecting the radius, the included angle shall be the greatest possible.

4. Required the linear aperture of a concave spherical reflector of glass, that the brightness of the sun's image may be the same when viewed in the reflector and in a given glass lens of the same radius.

5. Determine the evolute to the logarithmic spiral.

6. Prove that the periodic times in all ellipses about the same center are equal.

7. The distance of a small rectilinear object from the eye being given, compare its apparent magni-

tude when viewed through a cylindrical body of water with that perceived by the naked eye.

8. Find the fluents of the quantities $\frac{d\dot{x}}{x(a^2-x^2)}$, and $\frac{h\dot{y}}{y(a+y)^{\frac{3}{2}}}$.

9. Through what space must a body fall internally, towards the centre of an ellipse, to acquire the velocity in the curve?

10. Find the principal focus of a globule of water placed in air.

11. Determine, after Newton's manner, the law of the force acting perpendicular to the base, by which a body may describe a common cycloid.

12. Find the area of the curve whose equation is $xy=a^x$.

13. What is the value of the quantity q so that force × (period)$^2=q$ × radius of circle?

14. Two places, A and B, are so situated that when the sun is in the northern tropic it rises an hour sooner at A than at B; and when the sun is in the southern tropic it rises an hour later at A than at B. Required the latitudes of the places.

15. From what point in the periphery of an ellipse may an elastic body be so projected as to return to the same point, after three successive reflections at the curve, having in its course described a parallelogram?

Afternoon Problems.—Mr. Dealtry.

THIRD AND FOURTH CLASSES.

1. Inscribe the greatest cylinder in a given sphere.

2. Rays, which pass through a globe at equal distances from the centre, are turned equally out of their course.—Required a proof.

3. Given the declination of the sun and the latitude of the place, to find the duration of twilight.

4. A cylindrical vessel, 16 feet high, empties itself in four hours by a hole in the bottom.—What space does the surface describe in each hour?

5. Prove that if two circles touch each other externally, and parallel diameters be drawn, the straight lines, which join the opposite extremities of these diameters, will pass through the point of contact.

6. A ball, whose elasticity : perfect elasticity :: n : 1, falls from a given height upon a hard plane, and rebounds continually till its whole motion is lost.—Find the space passed over.

7. If a body revolves in any curve, compare the angular velocity of the perpendicular with that of the distance.

8. How far must a body fall externally to acquire the velocity in a circle, the force varying as the distance?

9. Given the right ascensions and declinations of two stars, to find their distance.

10. Find the velocity, with which air rushes into an exhausted receiver.

11. Let the roots of the equation $x^3-px^2+qx-r=0$ be a, b and c, to transform it into another, whose roots are a^2, b^2, c^2.

12. Find the fluent $\frac{z\dot{z}}{1+2az+z^2}$ where a is less than 1; and of $\frac{x\dot{x}}{\sqrt{a^2+x^2}}$.

13. Find that point in the ellipse, where the velocity is a geometric mean between the greatest and least velocities, the force varying $\frac{1}{D^2}$.

14. Determine the position of a line drawn from a given point to a given inclined plane, through which the body will fall in the same time as through the given plane.

15. The equation $x^3-5x^2+8x-4=0$ has two equal roots.—Find them.

16. Find the sum of the cube numbers $1+8+27+$&c. by the differential method; and sum the following series by the method of increments:

$1.2+2.3+3.4+$ &c. n terms.

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}$ &c. n terms and ad infinitum.

17. If half of the earth were taken off by the impulse of a comet, what change would be produced in the moon's orbit?

18. Prove that if the eye be placed in the principal focus of a lens, the image of a given object would always appear the same.

19. Find the time of emptying a given paraboloid by a hole made in the vertex.

20. Find the proportion between the centripetal and centrifugal forces in a curve; and apply the expression to the reciprocal spiral.

Afternoon Problems.—Mr. Dealtry.

FIFTH AND SIXTH CLASSES.

1. Prove that an arithmetic mean is greater than a geometric.

2. Every section of a sphere is a circle.—Required a proof.

3. If $\frac{3}{4}$ of an ell of Holland cost $\frac{1}{4}$£. what will $12\frac{2}{3}$ ells cost?

4. Prove the method of compleating the square in a quadratic equation.

5. Take away the second term of the equation $x^2 - 12x + 5 = 0$.

6. Inscribe the greatest rectangle in a given circle.

7. Sum the following series:

$$1 + 3 + 5 + 7 + \&c. \text{ to } n \text{ terms.}$$

$$3 - 1 + \frac{1}{3} - \frac{1}{9} + \&c. \text{ ad inf.}$$

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} \&c. \text{ ad inf.}$$

8. Find the value of x in the following equations:

$$\frac{42x}{x-2} = \frac{35x}{x-3}$$

$$\frac{x+1}{2} + \frac{x+2}{3} = 16 - \frac{x+3}{4}$$

$$3x^2 - 14x + 15 = 0.$$

9. In a given circle to inscribe an equilateral triangle.

10. Two equal bodies move at the same instant from the same extremity of the diameter of a circle with equal velocities in opposite semi-circles. Re-

quired the path described by the centre of gravity; find the path also when the bodies are unequal.

11. Through what chord of a circle must a body fall to acquire half the velocity gained by falling through the diameter?

12. Given the latitude of the place and the sun's meridian altitude, to find the declination.

13. Given the sun's altitude and azimuth and the latitude of the place, to find the declination and the hour of the day.

14. Prove that the velocity in a parabola : velocity in a circle at the same distance :: $\sqrt{2}$: 1.

15. How far must a body fall internally to acquire the velocity in a circle, the force varying $\frac{1}{D^2}$?

Evening Problems.—Mr. Dealtry.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. Find four geometric means between 1 and 32, and three arithmetic means between 1 and 11.

2. Suppose a straight lever has some weight, and at one end a weight is suspended equal to that of the lever; where must the fulcrum be placed, that there may be an equilibrium?

3. Determine the latitude of the place, where the sun's meridian altitude is 73°. 24′. 13″. its declination south being 16°. 36′. 47″.

4. If Q represent the length of a quadrant, whose radius is R, and the force vary $\frac{1}{D^2}$, the time

of descent half way to the center of force : the time through the remaining half :: $Q+R : Q-R$. Required a proof.

5. P and W represent two weights hung over a fixt pulley ; supposing P to descend, what space will it describe in t'', the inertia of the pulley being taken into the account ?

6. If a pendulum, whose length is 40 inches would oscillate in 1″ at the pole of a sphere, the radius of which is 4000 miles ; what must be the time of rotation round its axis, that the same pendulum at the equator may oscillate twice in 3″?

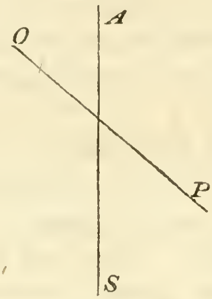
7. A given cone is immersed in water with its vertex downward ; what part of the axis will be immersed, if the specific gravity of the fluid : that of the cone :: 8 : 1 ?

8. The axis of a wheel and axle is placed in a horizontal position, and a weight y, which is applied to the circumference of the axle is raised by the application of a given moving force p applied to the circumference of the wheel ; given the radii of the wheel and axle, it is required to assign the quantity y, when the moment generated in it in a given time is a maximum, the inertia of the wheel and axle not being considered.

9. Would Venus ever appear retrograde according to the Tychonic system ?

10. A perfectly elastic ball begins to fall from a given distance SA in a right line towards the center of force S, the force varying $\frac{1}{D^2}$; in its descent, it impinges upon a hard plane OP inclined to SA at

a given angle, and after describing a certain curve comes to the plane on the other side, and is then reflected to the center; find the nature of this curve; and determine the whole time of descent to the center S in terms of the periodic time of a body revolving in a circle at the distance SA.



11. Let parallel rays be refracted through two contiguous double convex lenses; find the focal length on the supposition that the radii of all the surfaces are equal, and the sine of incidence : sine of refraction :: 5 : 4.

12. Given the latitude of the place and the declination of the sun, the former being less than the latter; to find at what time of the day the shadow of a stick would be stationary, and how far it would afterwards recede on the horizontal plane.

13. Transform the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$ into one, whose roots are the reciprocals of the sum of every $n-1$ roots of the original equation.

14. A body descends down the cycloidal arc AM, the base AL being parallel to the horizon and M the lowest point of the cycloid; determine that

point where its velocity in a direction perpendicular to the horizon is a maximum.

15. Construct the equation $a^2y - x^2y - a^3 = 0$.

16. Compare the time of descent to the center in the logarithmic spiral with the periodic time in a circle, whose radius is equal to the distance from which the body is projected downward.

17. Given the difference of altitudes of two stars, which are upon the meridian at the same time, and their difference of altitudes and difference of azimuths an hour afterwards, to find the latitude of the place.

18. A person's face in a reflecting concave decreases to the principal focus, and then increases in going from it.—Required a demonstration.

19. Prove that the mean quantity of the disturbing force of S upon P, in the 66th proposition of Newton, during one revolution of P round T, is ablatitious, and equal to half the mean addititious force.

20. The time of the sun's rising is the time which elapses between the appulse of the upper and under limb of the sun's disc to the horizon; given the sun's apparent diameter and the latitude of the place, it is required to determine the declination, when this time is a minimum.

21. Through a given point situate between two right lines given in position, to draw a third line cutting them in such a manner, that the rectangle under the parts intercepted between the point and the two lines may be a minimum.

22. Let a spherical body descend in a fluid from

rest; having given the diameter of the sphere, and its specific gravity with reference to that of the fluid, it is required to assign the velocity of the sphere at any given point of the space described.

23. The distance of the centre of gravity from the vertex of a solid formed by the rovolution of a curved surface is $\frac{2}{3}$ of its axis.—Determine the nature of the generating curve.

24. Suppose a given cylindrical vessel filled with water to revolve with a given angular velocity round its axis.—Required the quantity contained in the cylinder, when the water and cylinder are relatively at rest.

25. Sum the following series:

$\frac{10}{1.2.3.4} + \frac{14}{2.3.4.5} + \frac{18}{3.4.5.6} +$ &c. to n terms and ad inf.

$\frac{5}{1.2.3} \times \frac{1}{2^2} + \frac{6}{2.3.4} \times \frac{1}{2^3} + \frac{7}{3.4.5} \times \frac{1}{2^4} +$ &c. ad inf.

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \text{\&c. ad inf.}$$

26. Given the fluent $(a+cz^n)^m \times z^{pn+n-1}\dot{z}$ to find the fluent $(a+cz^n)^{m+1} \times z^{pn-1}\dot{z}$;

Required also fluent $\frac{\dot{x}\sqrt{a^2+x^2}}{x^3}$; and of $\frac{z^\theta \dot{z}}{1+mz}$, θ being a whole positive number.

Morning Problems.—Mr. Dealtry.

FIRST AND SECOND CLASSES.

1. Inscribe the greatest cone in a given spheroid.

2. A parabolic surface is immersed vertically in a fluid, whose density increases as the depth, with its base contiguous to the surface of the fluid; find upon which of the ordinates to the axis there is the greatest pressure.

3. Solve the equation $x^3 - px^2 + qx - r = 0$, whose roots are in geometric progression.

4. Suppose the reflecting curve to be a circular arc, and the focus of incident rays in the circumference of the circle, to find the nature of the caustic.

5. If the sine of incidence : sine of refraction :: $m : n$, required the focal length of a hemisphere, the rays falling first on the convex side.

6. If the subtangent of a logarithmic curve be equal to the subtangent of the reciprocal spiral, prove that the arc intercepted between any two rays in the spiral is equal to the arc intercepted between any two ordinates of the curve respectively equal to the former.

7. In what direction must a body be projected from the top of a given tower with a given velocity, so that it may fall upon the horizontal plane at the greatest distance possible from the bottom of the tower?

8. Draw an asymptote to the elliptic spiral.

9. If water or any fluid ascends and descends with a reciprocal motion in the legs of a cylindrical canal inclined at any angle, to find the length of a pendulum which will vibrate in the same time with the fluid.

10. Find the fluent $vx\dot{x}$, where $v =$ hyp. log. $(x + \sqrt{x^2 + a^2})$.

11. The centrifugal force at the equator arising from the rotation of the earth round its axis : the centrifugal force in any parallel of latitude :: (rad.)^2 : (sine.)^2 of the co-latitude.—Required a proof.

12. Given the latitudes of two places together with their difference of longitudes, to find the declination of the sun, when it sets to the two places at the same time.

13. Required the equation to a curve, whose subtangent is equal to n times its abscissa.

14. If the force vary $\frac{1}{D^{n+1}}$, how far must a body fall externally to acquire the velocity in any curve, whose chord of curvature at the point of projection is c? and apply the expression to the parabola and logarithmic spiral.

Afternoon Problems.—Mr. Palmer.

THIRD AND FOURTH CLASSES.

1. Find the value of £123333, &c.

2. Determine geometrically a mean proportional between the sum and difference of two given straight lines.

3. What is the general form of parallelograms, whose diameters cut each other at right angles?

4. Investigate the area of a circle, whose diameter is unity; and prove that the areas of different circles are in a duplicate ratio of their diameters.

5. Divide a given line into two parts, such that their product multiplied by their difference may be a maximum.

6. Prove that in any curve the velocity : velocity in a circle at the same distance (SP) :: $\sqrt{\text{chord of curvature}}$: $\sqrt{2SP}$.

7. A body projected from one extremity of the diameter of a circle, at an angle of 45^0, strikes a mark placed in the center. Required the velocity of projection and greatest altitude.

8. Find the area of a curve whose equation is $y = \dfrac{a^3}{a^2 - x^2}$.

9. In how many years will the interest due upon £100. be equal to the principal, allowing compound interest?

10. Admitting the periods of the different planets to be in a sesquiplicate ratio of the principal axes of their orbits, shew that they are attracted towards the sun by forces reciprocally proportional to the squares of their several distances from it.

11. Prove that in the course of the year the sun is as long above the horizon of any place as he is below it.

12. Determine the limits within which an eclipse of the sun or moon may be expected; and shew what is the greatest number of both which can happen in one year.

13. Prove that the time in which any regular vessel will freely empty itself : time in which a body will freely fall down twice its height :: area of base : area of orifice.

14. Find the fluents of $\frac{x\dot{x}}{\sqrt{a-x}}$; $\frac{x^2\dot{x}}{a-x}$.

15. Find the principal focus of a lens; and shew how an object may be placed before a double convex lens, that its image may be inverted and magnified so as to be twice as great as the object.

16. Prove that Cardan's rule fails unless two roots of the proposed cubic be impossible; and determine whether that rule be applicable to the equation $x^3 - 237x - 884 = 0$.

17. Deduce Newton's general expression in Sect. 9. for the force in the moveable orbit.

18. Define logarithms, and explain their use; also, prove that log. $A \times B =$ log. $A +$ log. B.

19. Explain the different kinds of parallax; and shew from the want of parallax in the fixed stars, that their distance from the earth bears no finite ratio to that of the sun.

Afternoon Problems.—Mr. Palmer.

FIFTH AND SIXTH CLASSES.

1. How many yards of cloth, worth 3*s.* $7\frac{1}{2}$*d.* per yard, must be given in exchange for $935\frac{1}{2}$ yards, worth 18*s.* $1\frac{1}{2}$*d.* per yard?

2. Find the interest of £873. 15*s.* 0*d.* for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ per cent.

3. Prove that the diameters of a square bisect each other at right angles.

4. Prove the opposite angles of a quadrilateral figure inscribed in a circle equal to two right angles.

5. Prove that if $A \propto B$ when C is given, and $A \propto C$ when B is given, when neither B nor C is given, $A \propto BC$.

6. Prove radius a mean proportional between tangent and cotangent; and that sine × cosine ∝ (sine)^2 of twice the angle.

7. Given the sine of an angle, to find the sine of twice that angle.

8. Prove that in the parabola (ordinate)^2 = abscissa × parameter.

9. Extract the square root of $a^3 - x^3$.

10. Solve the equation $3x^2 - 19x + 16 = 0$.

11. Prove that motion when estimated in a given direction is not increased by resolution.

12. Find the ratio of $P : W$ when every string in a system of pullies is fastened to the weight.

13. Prove that time of oscillation $\propto \frac{\sqrt{\text{length}}}{\sqrt{\text{force}}}$.

14. Prove that when a fluid passes through pipes kept constantly full, velocity $\propto \frac{1}{\text{area of section}}$.

15. Define the center of a lens; and find the center of a meniscus.

16. Find the fluxion of $\sqrt{a^3 + x^3 - \sqrt{a^2 - x^2}}$.

17. Prove elevation of the equator above the horizon = co-latitude.

18. Prove that sagita ∝ (arc)^2.

19. Prove that in the same orbit velocity $\propto \frac{1}{\text{perp.}}$.

Evening Problems.—Mr. Palmer.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. When £100. stock may be purchased in the 3 per cents. for £59½, at what rate may the same quantity of stock be purchased in the 5 per cents. with equal advantage?

2. A ball of wood being balanced in air by the same weight of iron, how will the equilibrium be affected when the bodies are weighed in vacuo? and by what weight of wood, properly disposed, may the equilibrium be restored?

3. Investigate the value of the circumference of a circle whose radius is unity.

4. Compare the areas of the parabolas described by two bodies projected together from the same point, and with the same velocity, towards a mark situated in an horizontal plane, the angles of elevation being to each other :: 2 : 1.

5. Prove the rule for finding the quadratic divisors of any equation; and apply it to the equation $x^4 - 17x^3 + 88x^2 - 172x + 112 = 0$.

6. On what point of the compass does the sun rise to those who live under the equinoctial, when he is in the northern tropic?

7. How many equal circles may be placed around another circle of the same diameter, touching each other and the interior circle?

8. Determine the resistance of the medium in which a body by an uniform gravity may describe a parabolic orbit?

9. Prove that a body moving in the reciprocal spiral, approaches or leaves the center uniformly.

10. Find the velocity and time of flight of a body projected from one extremity of the base of an equilateral triangle, and in the direction of the side adjacent to that extremity towards an object placed in the other extremity of the base.

11. Define similar curves; and prove that conterminous arcs of such curves have their chords of curvature at the point of contact in a given ratio.

12. Compare the time of a revolution about the center of a given ellipse, with that about its focus.

13. Find the attraction of a corpuscle placed in the axis of a cylindrical superficies, whose particles attract in an inverse duplicate ratio of the distance.

14. Prove that if the center of oscillation of a pendulum be made the point of suspension, the former point of suspension becomes the center of oscillation.

15. Determine the content of the solid generated by a semicircle revolving about a tangent parallel to it's base.

16. Find the fluents of

$$\frac{x^{-2}\dot{x}}{\sqrt{a-x}};\ y^{\frac{3n}{2}-1}\dot{y}\sqrt{a^n-y^n};\ \frac{v\dot{v}}{(a+v).(a^2+v^2)}.$$

17. Sum the series $1-\frac{1}{2^3}+\frac{1}{2^5}-\frac{1}{2^7}+$ &c. ad inf. and also to n terms.

$\frac{1}{1.5}+\frac{1}{1.6}+\frac{1}{3.7}+$ &c. to n terms. $\frac{1}{1.3}+\frac{1}{3.7}+\frac{1}{5.11}$ + &c. ad inf.

18. Required the sun's place in the ecliptic,

when the increment of his declination is equal to that of his right ascension.

19. Prove that the force by which a body may describe a curve, whose ordinates are parallel, is proportioned to $\pm \ddot{y}$; and determine the quantity q such that force $= q \times \pm \ddot{y}$.

20. Compare the times in which a cylinder, whose axis is parallel to the horizon, will discharge the first and last half of its content through an orifice in its lowest section.

21. Prove that the image of a straight line immersed in water appears concave to an eye placed any where between the extremities of the line.

22. At what distance from the earth would the apparent brightness of the moon be equal to that of Saturn and his ring together, supposing the apparent brightness of Saturn to that of his ring :: 2 : 1?

1803.

Morning Problems.—Mr. Hornbuckle.

FIRST AND SECOND CLASSES.

1. Find six geometric means between the numbers a and b.

2. Half the breadth of any very thin glass lens is a mean proportional between its thickness and focal length nearly. The demonstration is required.

3. Find the integral of $\dot{x}_4$, x being unity.

4. Sum the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$ &c. from the series expressing the cosine of a given arc in terms of the arc.

5. In what part of the moon's orbit is her gravity towards the earth unaffected by the action of the sun?

6. A globe of given weight is supported between two planes inclined to the horizon at the respective angles of 60^0 and 30^0. Compare the weights sustained by the planes with each other and with the whole weight.

7. Shew that a particle in the vertex of a triangle, from which a perpendicular falls on the base, is attracted towards the segments of the base with forces which are to each other inversely as the secants of the angles at the base; the law of attraction being the inverse square of the distance.

8. Two stars, whose right ascensions and declinations are given, are on the same azimuth, and the altitude of one of them is known; the latitude of the place is required.

9. Determine geometrically those points in an ellipse in which the centripetal and centrifugal forces are equal, the center of force being in the center of the ellipse.

10. Find the time of vibration of a cylindrical rod of given length suspended at one end, the density of which varies as the distance from the point of suspension.

11. Into how many parts must a given quantity, Q, be divided, that their continued product may be a maximum?

12. The altitude of the mercury in the gauge of a receiver is a inches, the standard altitude being h inches, after n turns; compare the capacities of the receiver and the barrel.

13. A ring of given weight descends by its gravity down the arc of a given quadrant which revolves uniformly about its axis perpendicular to the horizon in t''. Find the velocity of the ring at any point of its descent.

Afternoon Problems.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. Of right-angled triangles having the same base, determine that, down whose hypothenuse a body descends in least time.

2. Compare the angular velocities of SP and Sy, when a body moves in an ellipse, force in focus.

3. What is the space through which a sphere sinks in water; the specific gravity of sphere being $\frac{3}{4}$ of specific gravity of water?

4. From what arguments is it inferred that the earth's form is not spherical?

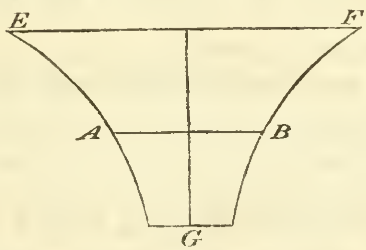
5. Required length of pendulum vibrating 180 times in a minute.

6. A and B hang over a pully, and $A = 2B$; through what space will a body descend by the action of gravity, whilst A descends two feet?

7. To what height would a body moving in an ellipse round the center ascend, if projected upwards in the direction of a line from the center, with the velocity with which it revolves?

8. Explain the construction of the common pump.

9. What must be the form of the vessel EFG, so that the pressure on every horizontal section, as AB, shall be the same?



10. If parallel rays pass through a prism whose refracting angle is small, and the angle of incidence be also small, shew that the deviation is proportional to the refracting angle.

11. Find the expression for the force of the body in the moveable orbit, supposing the immoveable orbit to be an ellipse with force in centre.

12. The focal length of a sphere is n times the radius of the sphere. Required proportion of sines of incidence and refraction.

13. Draw a line to touch two given circles.

14. Find the area of an equilateral and equiangular octagon.

15. If a globe in its quantity of matter = a cylinder that could exactly circumscribe it, what is the proportion between the densities of the globe and cylinder?

16. Given the height (h) and radius (r) of a section of the outer surface of a cylindrical shell, to find the thickness of the shell, when its quantity of matter = quantity of matter in sphere (rad r), (densities the same); shew also, than what line, h must necessarily be greater.

17. Give the definition of the fluxion of a quantity, and, from such definition, find the fluxion of $x^{\frac{m}{n}}$.

18. Find area of curve, the ordinate (y) of which $= ax + \frac{b}{\sqrt{1-x^2}}$.

19. Sum n terms of $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64}$, &c.

20. Shew how the logarithm of 101 may be found.

21. Which is greater, $x^3 + y^3$, or $x^2y + y^2x$, $x \gt y$?

22. If from any point within an equilateral tri-

angle, perpendiculars be drawn to the three sides, their sum equals a perpendicular drawn from one of the angles on the opposite side. Required proof.

Evening Problems.—Mr. Woodhouse.

1. Two weights, $A=5$, $B=3$, hang over a pully: What must be the length of a pendulum that makes one oscillation, whilst A descends one foot?

2. In a cylinder filled with water to the height (h), place a sphere, whose specific gravity $=\frac{1}{3}$ of the specific gravity of water; and whose diameter very nearly equals that of the cylinder: What will be the ascent of the water?

3. Given the horizontal refraction (h), find how much the rising of the sun is accelerated by it.

4. Let three points with velocities, V, V', V'', move the same way, uniformly in the periphery of a circle. Required the time of their conjunction, supposing them to quit a given point at the same time.

5. Sun's meridian altitude on longest day $=63^{\circ}$. $14'$. Required latitude of place.

6. Determine the form of a surface, such, that rays issuing from a given point, may, after refraction, be made convergent to another given point.

7. Let a body be projected obliquely to the distance, with a force varying as n^{th} power of the distance: shew in what values of n the angle between the apsides will not be affected by the excentricity of the orbit.

8. A funipendulous body in an uniformly resisting medium is accelerated by a force varying as the distance from the lowest point in the curve. Required the point where the velocity is the greatest; and also, the time of one oscillation.

9. Given the apparent diameter of the sun at the mean distance, and the angular distance from apogee; to find the sun's apparent diameter.

10. A body with a velocity of 80 feet per second is projected upwards, along a plane 50 feet long, and inclined to the horizon at an angle of 30^0: prove that the body, after quitting the plane, describes a parabola; and find the parameter.

11. The length of the solar year being $365^d. 5^h. 48' 49''$. shew that the intercalations may be determined from a series of fractions, as $\frac{4}{1}, \frac{29}{7}, \frac{33}{8}, \frac{128}{31}, \frac{161}{39}, \frac{2704}{655}, \frac{2865}{694}, \frac{5569}{1349}, \frac{86400}{20929}$; so that, the intercalation of 1 day in 4 years is too great, of 7 days in 29 years too little, of 8 days in 33 years too great, &c. &c.

12. If, on the inner surface of a paraboloid, a body whirled round as a circular pendulum (the point of suspension being in the axis of the paraboloid) describes a circle, then the time of revolution is the same, whatever is the circle, and equal to two oscillations of a common pendulum, the length of which equals the semi-parameter of the parabola. Required proof.

13. What sum ought to be given for the lease of an estate for 20 years, of the clear annual rent of

£100. in order that the purchaser may make 8 per cent. of his money?

14. $\text{Sin}.nA = 2\cos.A.\sin.(n-1)A - \sin.(n-2)A.$
$\cos.nA = 2\cos.A\cos.(n-1)A - \cos.(n-2)A.$
Required proof.

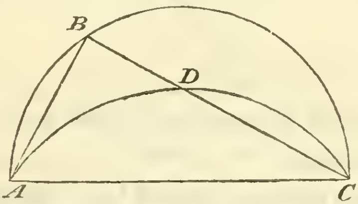
15. a, b, c, &c. being any quantities, make $\Delta a = b - a$, $\Delta^2 a = c - 2b + a$, $\Delta^3 a = d - 3c + 3b - a$, &c. then, $a + bx + cx^2 + \text{\&c.} = \frac{a}{1-x} + \frac{\Delta a.x}{(1-x)^2} + \frac{\Delta^2 a.x^2}{(1-x)^3}$ +&c. Required proof.

16. Find the integral equation of $ay + \frac{b\dot{y}}{\dot{x}} + \frac{c\ddot{y}}{\dot{x}^2} = 0$, a, b, c, being constant.

17. $(a+b)(a+b-1)(a+b-2)..(a+b-n+1)$
$= a(a-1)(a-2)...(a-n+1) + a(a-1)(a-2)$
$\ldots.(a-n+2)\,nb$
$+ a(a-1)(a-2)\ldots(a-n+3).\frac{n.n-1}{2}b.(b-1)$
+&c.
$+ b.(b-1)(b-2)\ldots.(b-n+1).$
Required proof.

18. If a cylinder and inscribed sphere be cut by two parallel planes perpendicularly to the axis, prove that the surfaces of cylinder and of sphere included between the planes, are equal.

19. ABC is a semicircle, ADC a quadrant:



prove that the line BA always $= BD$; and that the longer only of the lines AB, CB, can cut the quadrant.

20. Let a, b, c, be the sides of any rectilinear triangle; put $S=a+b+c$; then the area is equal to the square root of this product, viz.

$$\frac{S}{2}\cdot\left(\frac{S}{2}-a\right)\left(\frac{S}{2}-b\right)\left(\frac{S}{2}-c\right).$$

Required proof.

21. Let S', S'', S''', be the respective projections of any plane surface S, on the three co-ordinate planes, then $S^2=S'^2+S''^2+S'''^2$. Required proof.

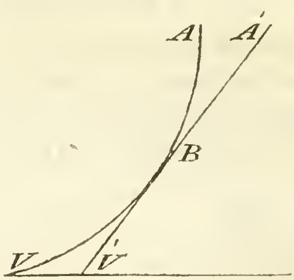
22. Shew that there can only be five regular polyhedrons.

23. Of all isoperimetrical polygons having the same number of sides, the greatest is that which is equilateral. Required proof.

Morning Problems.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. The straight line $A'BV'$ = the semi-cycloid ABV, and touches it at the middle point B: com-



pare the time of describing $A'BV'$ with the time of half an oscillation.

2. Let the reflecting curve be a circular arc, and let parallel rays be incident in its plane; find the

cusp of the caustic, and prove the density of reflected rays to be there greater, than at any other point.

3. Two bodies from different distances fall to the center of force $\left(\text{force} \propto \frac{1}{(\text{dist})^2}\right)$; then shew, that the times of their descent are in the sesquiplicate ratio of the whole spaces described.

4. Compare the times of emptying (on the common hypothesis) through equal orifices, two prisms of different heights and bases.

5. Prove that at the point where the $\angle SPy$ (in any curve) is a minimum, the angular velocities of SP and Sy are equal; and determine that point in an ellipse, force in focus.

6. If a body fall down the axis of a right-angled cone, the vertex downwards and axis vertical, in what point of its descent will it have acquired velocity sufficient to describe a circle on the surface of the cone, when whirled round as a circular pendulum?

7. The force varying as the distance, project a body from a given distance (d), at a given angle (a): investigate the orbit after the manner of the 8th section.

8. Transform $x - \frac{x^2}{2} + \frac{x^3}{3} - \&c.$ into a series as

$$\frac{ax}{1+x} + \frac{a'x^2}{(1+x)^2} + \frac{a''x^3}{(1+x)^3} + \&c.$$

9. Find a number, of which divided by 2, 3, 5, respectively, the remainders are 1, 2, 3.

10. Determine the length of the curve, in which

the rectangle of the abscissa and ordinate = the rectangle of the subtangent and a given line (a).

11. Shew the method by which Archimedes approximated to the length of the circle.

12. Of all triangles having the same base and perimeter, the isosceles triangle is that of which the area is the greatest. Required proof.

13. The surface and solidity of a sphere are respectively equal to two thirds of the surface and solidity of the circumscribing cylinder, the area of the bases of the cylinder forming part of its surface. Required demonstration.

14. Prove that the surface of a sphere can be completely covered with the surfaces either of 4, or of 8, or of 20 equilateral spherical triangles.

Afternoon Problems.—Mr. Hornbuckle.

THIRD AND FOURTH CLASSES.

1. In what time will the interest of £1. amount to 15*s.* allowing $4\frac{1}{2}$ per cent. simple interest?

2. Solve the following equations $\frac{16}{x} - \frac{100-9x}{4x^2} = 3$

$$\left.\begin{aligned} x^2y + y^2x &= 30 \\ \frac{1}{x} + \frac{1}{y} &= \frac{5}{6} \end{aligned}\right\}.$$

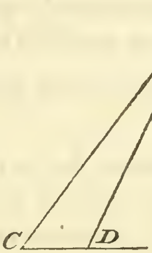
3. Extract the square root of $7 - 4\sqrt{3}$.

4. Find, by means of a given circle, two angles whose sines are in the proportion of 9 : 4.

5. Two bodies, A and a, are projected perpen-

dicularly upwards with the velocities V and v: How high will their common center of gravity ascend?

6. Determine that part of the inclined plane AC



through which a body will descend from rest in the time of describing AD.

7. Find the focal length of a meniscus whose thickness is inconsiderable.

8. The reflecting curve is the logarithmic spiral and the radiating point in the center. Construct the caustic.

9. At what angle must the rudder of a vessel be inclined to the stream that the effect produced may be a maximum?

10. Prove $\dot{z} = F\dot{x}$, z being the space fallen through by the uniform force, 1, to acquire the velocity.

11. Prove the differential method by the method of increments.

12. A cylindrical tube, 40 inches long, is half filled with mercury and then inverted: How high will the mercury stand, the standard altitude being supposed 30 inches?

13. Required the velocity with which the air will rush into an exhausted receiver.

14. Find the fluents of $x\dot{x}\sqrt{a^2 - x^2}$, $\frac{\dot{x}}{x\sqrt{x-a}}$, $a^x\dot{x}$.

15. Investigate Cardan's rule for the solution of a cubic equation, and shew that it fails if all the roots are possible.

16. Shew that the general equation to the apsides can have but four possible roots.

17. Shew where an inferior planet is a morning or evening star.

18. Prove that when Aries rises, the ecliptic makes the least angle with the horizon, and that this angle continually increases to its setting.

19. Render Newton's expressions for the law of force in his second section general.

20. If a body fall from a finite distance, and a given ellipse be described whose axis major is this distance, prove that the area described by the radius vector SD is equal to the area described in the same time in a circle at the distance of half the latus rectum of the ellipse from the same center.

Evening Problems.—Mr. Hornbuckle.

1. Find the equated time of payment of two sums S and s, due at T and t years hence respectively, allowing simple interest.

2. A body is projected from the top of a tower of a given height, at a given angle of elevation, and with a given velocity : find the range described on a plane drawn from the base of the tower at any given angle of inclination to the horizon.

3. The circumference of a given circle revolves uniformly about its diameter with a given angular velocity : compare the momentum of the circumference with that of those sides of the circumscribed square which are parallel to the axis of motion.

4. Suppose the axis major of a spheroid : the distance between the foci :: sine of incidence : sine of refraction, and that rays parallel to the major axis are incident on the convex surface; at that point in which any ray is incident draw a tangent to the ellipse, and a perpendicular to the tangent cutting the axis major; also join the point of incidence and each of the foci. It is required to prove, without any other construction, that the ray will be accurately refracted to the farther focus.

5. Sum the following series:

$$\frac{1}{1.2}\times\frac{2}{3}-\frac{1}{2.3}\times\frac{3}{4}+\frac{1}{3.4}\times\frac{4}{5}-\frac{1}{4.5}\times\frac{5}{6}+\text{\&c. ad inf.}$$

and $\frac{4}{1.2.5}+\frac{5}{2.3.6}+\frac{6}{3.4.7}+$ &c. to n terms and to infinity.

6. Given the apparent perpendicular depth of an object under the water, to find the direction in which a ball must be fired from a given situation to hit it, the path of the ball being supposed rectilinear.

7. Investigate the fluxional expression for the content of a solid, and apply it to find the content of that, which is generated by the revolution of a curve about its axis, whose equation is $y=$ hyp. log.

$(1+x)$, x being the abscissa, and y the corresponding ordinate.

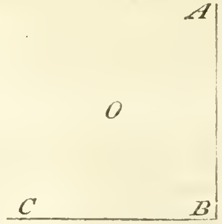
8. Prove that the sum of the infinite series $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \&c. = \frac{Q^3}{4}$, Q being a quadrantal arc to radius 1.

9. The sum of the m^{th} powers of the roots of the equation $x^n - 1 = 0$ is n, if $\frac{m}{n}$ be any whole number; and 0, if it be a fraction. Required a proof.

10. Compare the space described in one second by the force of gravity in any given latitude with that which would be described in the same time, if the earth did not revolve round its axis.

11. At the extremity of the arms of a given lever are suspended respectively the weights p and q; supposing p to descend, and the lever to revolve round the fulcrum; it is required to determine the angular velocity of the lever at any point of its descent from an horizontal to a vertical position; the inertia of the lever being neglected.

12. Required the length of the longest ladder



that can be slided up a wall AB from the horizontal plane CB, under an obstacle O given in position.

13. Prove the diurnal path of the shadow of any object perpendicular to the horizon in all latitudes a straight line, when the sun is in the equinox.

14. The refracting curve is the logarithmic spiral, and the radiating point in the center. Construct the caustic.

15. A flexible string of given length is wrapped round a cylinder whose weight and dimensions are given; one end of it being fixed to the surface of the cylinder, and the other to a tack: if the cylinder be suffered to descend by its gravity, it is required to find the time in which the whole string is unwound, and the velocity acquired.

16. Given the force of gravity on the earth's surface; the periodic time of the moon, and her mean distance. It is required to compare the quantities of matter in the moon and earth.

17. Find the fluents of $\frac{a^2x - \frac{1}{2}\dot{x}}{(a+x)^{\frac{3}{2}}}$, $\frac{x^n\dot{x}}{1+ax+x^2}$, n being a whole number.

18. Construct a vertical east dial.

19. A vessel of given dimensions in the shape of a paraboloid is filled with water, and revolves uniformly round an axis perpendicular to the horizon. Required the time of rotation that it may just be emptied.

20. Find the fluxional expression for the force tending in lines parallel to the axis of a curve, when the oscillations therein are isochronous; and apply it to the catenary.

21. A flag-staff of given height is erected on a tower whose height is also given: at what point on the horizon will the flag-staff appear under the greatest possible angle?

22. Find the pressure sustained by a given horizontal plane; the height of the atmosphere being infinite, and its density being supposed uniform and n times as great as that at the earth's surface.

1804.

Morning Problems.—Mr. HUDSON.

FIRST AND SECOND CLASSES.

1. POINT out the difference in principles, in exhaustions, indivisibles, prime and ultimate ratios, and the method of fluxions.

2. Find the integral of

$$(x-m).(x-5m)+\frac{(x-2m).(x-5m)}{2}, \text{ the}$$

increment of (x) being expressed by (m).

✓ 3. *A* and *B* play, by turns, 2 bowls with equal skill; *A* wants 3 of the game, and *B* 2; find their respective probabilities of winning.

4. Two elastic balls, beginning their motion from different points in the same right line perpendicular to an horizontal plane, are inflected along the plane with the velocities acquired, by a hard plane inclined to their direction at an angle of 45^0; given the distance at which one impinges on the other on the horizontal plane, and the point from which one of them descended, to find the point from which the other began its motion.

5. With a given straight line perpendicular to

the horizon, as radius, and the lower end as center, describe a circle, and draw a tangent at its extremity; if any radius of this circle represent the position of an inclined plane, and a body be projected on it with the velocity acquired in falling through the given line, the space described on this plane in the time of its descent will be equal to the radius of the circle, together with the perpendicular drawn from the intersection of the plane and the circle to the tangent.

6. Let the atmosphere be supposed to consist of equally elastic particles, which at different distances repel each other with forces inversely proportional to the squares of the distances between their centers, and let the tube of a barometer be partly filled with mercury, and then its open end immersed in a bason of the same fluid; investigate an equation for finding the height of the mercury in the tube.

7. At a certain place in a given north latitude, the sum of the sun's declination and altitude was observed at a given hour in the morning; find the sun's altitude and declination.

8. Let a small reflector be moved along an horizontal plane with its surface parallel to it, and let an object move in a straight line perpendicular to the plane, and so as to be always at the same distance as at first from the reflector; find the locus of an eye, which being always equidistant from the reflector, shall see the image.

9. Three known rectilinear objects placed contiguous, and in the same right line, appeared to the

eye of a spectator of the same length; find his position.

10. Let a body be projected at an angle of 45° with the horizon, and suppose the arc of the earth's surface intercepted between the points of projection and incidence to be 60°; find the velocity of projection.

11. Let the earth be supposed a sphere of given magnitude, and to revolve about its axis in a given time; compare the weight of a body at its equator with its weight in a given latitude.

12. Investigate the rule for finding by fluxions the center of oscillation of any line, surface, or solid.

13. Find the solid content of a section of a given spheroid made by a plane coincident with the ordinate of a given diameter, and cutting the diameter at a given distance from the center.

14. Find the locus of the vertex of a triangle described on a given base; 1. when the sum of the angles at the base is given: 2. when one of them is always double the other.

Morning Problems.—Mr. WOODHOUSE.

FIRST AND SECOND CLASSES.

1. Required two geometric means between 10, and 100.

2. $b = \frac{a - \sqrt{(a^2 - x^2)}}{a + \sqrt{(a^2 - x^2)}}$. Required x.

3. Prove that tang. $\frac{A}{2} = \frac{\sin. A.}{1+\cos. A}$ (radius 1).

4. Prove that if the difference between the digits of the odd and even places in any number be adjoined to the number, the result is divisible by 11.

5. Suppose a square and its circumscribing circle to revolve round a diagonal of the square; compare the surfaces and solidities of the solids generated.

6. Shew how the circumference of an ellipse may be computed, of which the excentricity is .99, the semi-axis major being 1.

7. Given the force at earth's surface, earth's radius, and moon's distance, to find the moon's periodic time.

8. Find the fluent of

$$\frac{x^2\dot{x}}{(1+x^2)^{\frac{3}{2}}}; \text{ and prove } \int \frac{x^5\dot{x}}{\sqrt{(1-x^2)}},$$

contained between the values of x, 0 and 1, to equal $\frac{2.4}{3.5}$.

9. Explain the principle on which achromatic telescopes are constructed.

10. If parallel rays fall on equal apertures of spherical reflectors, the lateral aberrations vary inversely as the squares of the radii of the reflectors. Required proof.

11. What sum of money ought to be paid every three years instead of £100. paid annually, the rate of interest being 5 per cent.?

12. Shew that the rectification of the hyperbola may be deduced from that of the ellipse.

13. Shew that if the moon's velocity were increased, her synodic revolution would differ less from her sidereal.

14, In the lemniscata, whose equation is $(x^2+y^2)^2 = y^2 - z^2$, it is required to assign two arcs equal to one another.

Afternoon Problems.—Mr. Hudson.

THIRD AND FOURTH CLASSES.

1. In a system of notation whose local value is (a), in any multiple of $(a-1)$ the sum of the digits is either equal to $(a-1)$, or a multiple of it.

2. Any power of a cube root of unity, is itself a cube root of unity.

3. If two circles touch each other, any straight line passing through the point of contact cuts off similar parts of their circumferences.

4. If from the angular points of any plane triangle, three straight lines be drawn through the same point within it to the sides, 6 segments will be formed; shew that the solids, measured by 3 that are not contiguous, are equal.

5. Given the tangents of two arcs of a given circle, to find the tangents of the arcs which are equal to their sum, and difference.

6. A person wishing to determine the height of an obelisk standing on a declivity, measured a given

distance from its base, and took its elevation; and going on in the same direction a given distance, took its elevation again; shew how the altitude may be found from these data.

7. Given the perimeter, the vertical angle, and the perpendicular in a plane triangle, to construct it.

8. Solve the equations,

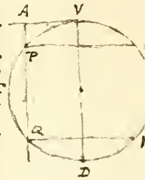
1. $3x^{\frac{5}{3}} + x^{\frac{5}{6}} = 3104.$

2. $x^3 - (a+p).x^2 + .(ap+q).x - aq = 0.$

3. $x^2 + xy = 10,$
$y^2 + yz = 21,$
$z^2 + xz = 24.$

9. The evolute of a common cycloid is another equal cycloid.

10. On a given straight line perpendicular to the horizon, describe a circle, and draw a tangent at its higher extremity; if any point be assumed in this tangent, and a straight line be drawn from it parallel to the diameter, cutting the circle in two points, and chords be drawn from those points perpendicular to the diameter, the times of describing the parts of this line, together with the times of describing the corresponding chords with the velocities acquired at their extremities, shall be equal.



11. Let the compressive force be supposed proportional to the square of the density, and the force of gravity inversely proportional to the square of the distance from the earth's center; it is required to find the law of the density in the atmosphere.

12. Given the azimuth and altitude of the sun at two different times during the same morning, to determine the latitude of the place.

13. Let the radii of surfaces of a double convex lens of known refracting power be given, and the positions of the eye and object in the axis of the lens; to compare the apparent magnitudes of the object and image.

14. To an eye that moves in the circumference of a given circle, find where a known rectilinear object given in position in the same plane with the circle will appear a maximum, and where a minimum.

15. Let a weight appended at the circumference of a wheel elevate by its gravity a weight appended at the axle; given the two weights, the weight of the wheel, and the radius of the axle, to find at what distance one of the weights must be applied, so as to raise the other at the axle through a given space in the least time.

16. A body having descended from a given altitude, begins to descend along the arc of a given cycloid whose base coincides with an horizontal plane; find the points where it will leave the curve, and where it will meet the horizontal plane.

17. Find the ratio of the times of oscillation of a pendulum, at the equator, and at the pole, supposing the earth to be a sphere, and to revolve about its axis in a given time.

18. If the first fluxions of a common ordinate of a curve and a circle be equal, they will have the same tangent at that point; and if the second fluxions be equal, it will be impossible to describe another circle that shall approach nearer to the curve.

19. If (x) increase in geometrical progression, the successive increments of the ratio of (x) to (1) will be equal.

20. Of all cones, of the same solid content and specific gravity, to determine the ratio of the diameter of the base to the altitude in that, which, when immersed with its axis perpendicular to the surface of a given fluid, shall have the surface immersed, a minimum.

21. A body revolving in a given circle, acted on by a force, which is inversely proportional to the square of the distance from the center, in consequence of an impulse in the direction of its motion, begins to describe an orbit of given excentricity; find the velocity communicated to it by the impulse.

22. The given circle described by a body, acted on by a force tending to a point in the periphery, is caused to revolve about the center of force in such a manner that the angular velocity of the body describing it, is to that of a body describing the same orbit when quiescent, in a given ratio; find the equation to the orbit described in fixed space, the law of force, and velocity of projection at the apsis, by which a body might describe it.

23. Find the point in the moon's orbit, which is here supposed circular, when that part of the ablatitious force which accelerates her motion is a maximum.

24. If two bodies attract each other with forces inversely proportional to the square of the distance between their centers, and be projected in parallel and opposite directions with given velocities, they

will describe about their common center of gravity similar figures.

Afternoon Problems.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. The interior angles of every quadrilateral figure are equal to four right angles. Required proof.

2. Reduce .855 of a foot, to the decimal of a yard.

3. Given the logarithms of 2 and 3, to find the logarithm of 216.

4. What three numbers are those that have their differences equal, their sum 15, and sum of cubes 495?

5. Sum $\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+$ &c.

6. Prove that $\sin.(a+b).\sin.(a-b)=(\sin.a)^2-(\sin.b)^2$.

7. Shew how the focus of a concave lens may practically be found.

8. Explain the principle and use of the hydrostatic balance, and determine the specific gravity of a piece of silver that weighs 135 grains in air, and 123.3125 in water.

9. Required center of gravity of three equal bodies placed at the three angles of an isosceles right-angled triangle.

10. Shew that there is no finite distance through

which a body at rest can fall, so as to acquire the velocity in the logarithmic spiral, the force $\propto \frac{1}{D^3}$.

11. Let a ray of light fall obliquely on a plane refracting surface; determine the position of the refracted ray, the ratio of the sines of incidence and refraction being given.

12. From what arguments is it probable that the moon has no atmosphere?

13. Shew the variation of the earth's angular velocity to be nearly twice as great as it would have been, had the earth's motion been uniform.

14. Compare the apparent diameters of the sun at mean and least distance.

15. Two bodies projected along two planes inclined to the horizon at angles 45^0, and 30^0, describe spaces respectively as $\sqrt{2} : \sqrt{3}$. Required the ratio of the initial velocities of the projected bodies.

16. Let two equal bodies from the vertex of an isosceles triangle, the vertical angle being 120^0, move uniformly along the sides; compare the space described by the center of gravity with the space described by either body.

17. Compare the apparent diameters of the sun seen from the earth and Jupiter.

18. In 39th proposition find the area *ATVME* when $DF = a$, a being a given line.

19. What would be the proportion between the axes of the earth's orbit, if the apparent diameter of the sun at mean distance were to his apparent diameter at perigee :: 2 : 5?

Afternoon Problems.—Mr. HUDSON.

FIFTH AND SIXTH CLASSES.

1. If the number (2) be divided into any two parts, the difference of the parts is equal to the difference of the numbers, formed by adding each to the square of the other.

2. Solve the equations,

1. $\frac{2x-3}{3} - \frac{10x-4}{18} = -1.$

2. $x^{\frac{6}{5}} + x^{\frac{3}{5}} = 756.$

3. $(x+2)^2 + (x+2) = 20.$

4. $x + y + \sqrt{xy} = 19.$
$x^2 + y^2 + xy = 133.$

3. The space described by a heavy body in the 4^{th} second, is to the space described in the last second except 4, as 1 to 3; required the whole space described.

4. Let a cubical vessel be filled, half with water, and half with mercury; find the ratio of the pressure on the sides to the pressure on the base.

5. Of all triangles described on the same base, and having the same altitude, the perimeter of the isosceles is least.

6. If the abscissa of an algebraic curve increase uniformly, the deflection from the tangent is ultimately equal to half the quantity that measures the second fluxion of the ordinate.

7. A body is projected at a given point with a given velocity, acted on by a force which varies

inversely as the square of the distance from the center; compare the chord of curvature at the point of projection with the distance.

8. A body moves through a given chord of a great circle of the earth in a tube, acted on by a force which is at every point proportional to its distance from the earth's center; find the ratio of the whole time of describing the tube, to the periodic time of a body revolving in a circle at the earth's surface.

9. Given the base, the vertical angle, and the perpendicular, in a plane triangular, to construct it.

Afternoon Problems.—Mr. Woodhouse.

FIFTH AND SIXTH CLASSES.

1. At $3\frac{1}{2}$ per cent. to find the interest of £3. 15*s.* 4*d.* for one year.

2. It is required to inscribe in a circle an equiangular and equilateral octagon.

3. Find the time of describing 30 feet on a plane inclined to the horizon at an angle of 30°, the force of gravity being supposed to be diminished by one-fourth of its present quantity.

4. Shew that the velocity of the earth continually increases from the mean to the perihelion distance.

5. Which is greater, $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+$ &c. ad inf.

or $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+$ &c. ad inf.?

6. Prove that $\cos. 2a = 2 (\cos. a)^2 - 1$ (radius $= 1$.)

7. Find the focal length of a plano-convex lens, the sine of incidence being to the sine of refraction :: 1,5 : 1.

8. Find the fluxion of $\frac{x}{\sqrt{(1+x^2)}}$, and the fluent of $\frac{\dot{y}}{(1+y^2)^{\frac{3}{2}}}$.

9. Compare the pressure on the inner surface of a cylinder filled with water, with the weight of the fluid.

10. The emersion of a satellite of Jupiter is observed to happen at a certain place at $11h.\ 30'$. apparent time, and at Greenwich it is calculated to happen at $10h.$; find the longitude of the place with respect to Greenwich.

Evening Problems.—Mr. Hudson.

FOR THE FIRST SIX CLASSES.

1. Solve the equations,

$$3x^{\frac{4}{3}} - \frac{5x^{\frac{8}{3}}}{2} = -592.$$

$$(x-4)^2 + 2.(x-4) = \frac{2}{x} - 1.$$ N.B.

2. Having given the sum and product of two algebraic quantities, investigate the series expressing the sum of their n^{th} powers.

3. Find the sum of the infinite series,

$$\frac{1}{1.2} - \frac{3}{4.5} + \frac{5}{7.8} - \frac{7}{10.11} \&c.$$

and n terms of the series,

$$2 + 6 + 14 + 30 + 62 + 126 \&c.$$

4. Find the following fluents,

$$\frac{a^2 \dot{x}}{(a+x).(a-x)^{\frac{3}{2}}}; \text{ and}$$

$$\frac{z\dot{x}}{\sqrt{(2rx - x^2)}};$$ where x = circular arc whose radius $= r$, and versed sine $= x$.

5. Solve the fluxional equation $\dot{x} + x^3\dot{x} = \dot{z} + z\dot{x}$.

6. A person holding two common dice, undertakes to throw both the numbers 5 and 7, at 3 trials; find the odds against him.

7. Find the center and diameter of a circle that touches 3 given circles, each of which touches the other two.

8. If a spheroid be cut by a plane, the section will either be an ellipse or a circle, and all parallel sections will be similar figures.

9. A person standing in the center of a given circle, throws an elastic ball with a given velocity along a plane coinciding with the horizontal radius, which is reflected to him by a small hard plane coinciding with the tangent; if the plane be conceived to revolve about the center of the circle, find in fixed space the locus of his position on the plane, when the ball projected with the same velo-

city, and reflected by the tangent-plane, always returns to his hand in the same time.

10. A ball half elastic having fallen from a given point, at the middle of its descent is projected parallel to the horizon with the velocity acquired; find the position of a hard plane, which, opposed perpendicularly to its motion, will cause it to move to the same point to which it would have descended in the right line, and find the whole time of its motion.

11. To determine the length of a pendulum which, beginning an entire oscillation in a common cycloid from a given point, shall arrive at a given perpendicular to the horizon in less time than any other, beginning an oscillation from the same point.

12. A given parabolic conoid is put into a fluid, and when it is in a position of permanent equilibrium the base is wholly extant, and inclined to the horizon; having given its specific gravity relatively to that of the fluid, it is required to determine the dimensions of the part immersed.

13. The solid described in the last question being excavated, and containing a fluid, has its axis inclined to an horizontal plane till the surface of the fluid passes through one extremity of its base; in this position a hole being made at the extremity of that diameter which passes through the center of the fluid's surface, and the greatest perpendicular distance of the surface from the base being given, it is required to determine the time in which it will be emptied.

14. At any place situated between the equator and the poles, it is required to determine the two hours in the morning, during which the increase of the sun's altitude is greatest and least, on that day when he is in the equinoctial point.

15. At a given place in north latitude, two stars, the difference of whose right-ascensions is known, are observed to be on the prime vertical at the same time; in this position their distance being observed, it is required to determine their declinations.

16. Given the sun's altitude and declination, and the sum of the azimuth and hour-angle, to determine the latitude of the place.

17. Let a ring of given diameter be held with its plane perpendicular to the surface of a large plane reflector, and let a straight line be drawn from its center perpendicular to its plane; having given the position of the eye in this line, it is required to determine the position and apparent magnitude of the image.

18. Let the surface of a plane reflector be always perpendicular to a line which revolves about one of its extremities, and cuts two other lines given in position; it is required to determine the locus of the reflector, so that an object moving in the intersection of the revolving line with one of the given lines, the image shall move in its intersection with the other.

19. Let the position of the axis of a spherical surface of known refracting power, perpendicular to,

and bisecting, a very distant object, be given, and in it the position of the eye and image, and also the apparent magnitudes of the object and image; to determine the magnitude and position of the refracting surface.

20. A body is projected in a given direction, at a known distance from an horizontal plane, with a given velocity, acted on by a force perpendicular to the plane, which is inversely proportional to the square of its distance from the plane; construct the orbit it will describe.

21. It is required to compare the polar and equatorial diameters:—1st, When the length of a degree of latitude, in each of two known places, is given. 2. When the length of a pendulum vibrating seconds, in each of the same two places, is given.

22. Two bodies whose weights are known, lying on a smooth horizontal plane, are connected by a flexible line passing through a small ring fixed at a given point between them; in this position a given velocity is communicated to one of them in a direction perpendicular to the line that joins their centers, and the other is made to move directly towards the ring. Investigate the motion of the projected body, and find the angle described when the other body arrives at the ring.

Evening Problems.—Mr. Woodhouse.

1. What is the interest of £315. 10*s.* for one year, at $4\frac{3}{4}$ per cent.?

2. Of all triangles that can be formed by two

given sides containing a variable angle, the greatest is that in which the contained angle is a right-angle.

3. Find the present value of £20. to be paid at the end of every five years, at 5 per cent. rate of interest.

4. Prove that

$$\frac{\sin.(a-b)}{\sin.a.\sin.b}+\frac{\sin.(b-c)}{\sin.b.\sin.c}+\frac{\sin.(c-a)}{\sin.c.\sin.a}=0.$$

5. If any triangular number be multiplied by 8, and increased by 1, the result is a square number. Required proof.

6. In a triangular pyramid, the sum of the squares of the faces that are perpendicular to each other, equals the square of the remaining face.

7. With two dice, compare the chance of throwing at one throw, an ace, with the chance of throwing five points.

8. If a body suspended by a string oscillates through a quadrant (the extremity of the quadrant being the lowest point,) the force stretching the string at the lowest point is three times that which is due to the weight of the body. Required proof.

9. In a circle, compare the time of falling down a chord of 60° to the extremity of a diameter perpendicular to the horizon, with the time of the oscillation of a pendulum equal in length to the chord.

10. Under an exhausted receiver a sphere sinks to a depth $= \frac{3}{4}$ diameter. Required the alteration in

the depth of the immersion, when the air (its specific gravity being 0.00122) is admitted.

11. Let parallel rays fall on a spherical refractor, determine by construction the position of the refracted ray, and shew that it cuts the axis in a point nearer the surface, than any other ray does less remote.

12. Let the distance between two thin and equal plano-convex glass lenses, be equal radius of convex surface, determine the focus of parallel rays after refraction at each lens.

13. The excentricity of the earth's orbit being small, the variation of the angular velocity is nearly proportional to the cosine of the angle made by the radius vector and perihelion distance. Required a proof.

14. From the poles to the equator, the decrease of the length of a pendulum always vibrating in the same time, varies as the square of the cosine of latitude. Required proof.

15. If the altitude of a wave compared with its breadth be small, the wave moves over a space equal to its breadth, nearly in the same time that a pendulum, equal in length to the breadth of the wave, performs one oscillation. Required proof.

16. In the astronomical telescope, shew that, with a given object-glass, the magnifying power is increased, by increasing the convexity of the eye-glass; and explain what circumstances confine the increase of the convexity within certain limits.

17. In an elliptical orbit, if a be the semi-axis major, e the excentricity, u the angle contained between the radius vector and perihelion distance, then

$r = \frac{a.(1-e^2)}{1+e.\cos. u}$. Required proof.

18. Find fluents of $\frac{\dot{x}}{x^3\sqrt{(1-x^2)}}$ and of $(\sin. z)^4\dot{z}$.

19. Prove that $\int\frac{\dot{x}}{\sqrt{(1-x^4)}}$ contained between the values of x, 0 and 1, =

$$\left(1-\frac{1}{2^2}+\frac{1^2.3^2}{2^2.4^2}-\frac{1^2.3^2.5^2}{2^2.4^2.6^2}+\&c.\right)\pi.$$

π being $= \frac{3.14159}{2}$.

20. $\sin. 3a + \sin. 5a + \sin. 7a + \&c. \sin.(2m-1)a$ $= \frac{\sin.(m+1)a.\sin.(m-1)a}{\sin. a}$. Required proof.

21. It is required to divide the quadrantal arc of an ellipse in such a manner, that the difference of the arcs may equal the difference of the semi-axes.

22. The sum of the squares of the coefficients of a binomial whose exponent is n, (n an integer,) equals the coefficient of the middle term of a binomial whose exponent is $2n$. Required proof.

23. Let two weights fastened to equal strings (l, l) move in a vertical plane, the one oscillating, the other revolving, and at the lowest points let the velocities due to heights h, h', be such, that $\frac{h}{2l}=\frac{2l}{h'}=m^2$, then

the time of the revolution of one weight is to the time of oscillation of the other as m to 1. Required proof.

24. The periodic times of planets revolving in elliptical orbits are independent of the excentricities of those orbits. Required proof.

1805.

First Morning.—Mr. HORNBUCKLE.

MONDAY, JANUARY 14, 1805.

1. THE discount on a promissory note of £100. payable a year and a half hence, amounted to £7. 10*s*. What interest per cent. did the banker make of his money?

2. Prove that a table of sines and cosines constructed for 45°, will be sufficient to exhibit the sine or cosine of any arc in the circle.

3. Divide a given arc (A) less than a quadrant into two such parts, that the secant of one multiplied into the square of the secant of the other may be a maximum.

4. Find the principal focus of a plano-convex glass lens, and prove it a fixed point.

5. Suppose $A =$ the sum of the roots of an equation, B the sum of the squares, C the sum of the cubes, &c. Required an expression equal to $A+B+C+$ &c.

6. In a given latitude, it is required to determine the declination of the sun, that it may set in the south west point.

7. Two bodies, whose weights are 4lb. and 3lb. are projected together at an angle of 45° to the

horizon, with the respective velocities of 19 and 12 feet in a second; required the path described by their common center of gravity, and its greatest height above the horizontal plane.

8. Find the present worth of an annuity of (A) pounds, to begin (n) years hence, and continue (t) years at compound interest.

9. The refracting curve is the logarithmic spiral, and the radiating point in the center; construct the caustic.

10. Given the velocity of the wind, the angular motion of the sails of a windmill, and their inclination; required the length of the arms, that the effect produced by the wind may be the greatest.

11. Sum the following series:

$$\frac{1.2}{2.3} - \frac{2.3}{3.4} + \frac{3.4}{4.5} - \frac{4.5}{5.6} + \&c.$$

and $$\frac{10}{1.2.3.4} + \frac{14}{2.3.4.5} + \frac{18}{3.4.5.6} \text{ ad inf.}$$

12. A given body impelled by the uniform force of gravity descends in a medium wherein the resistance varies as the velocity. It is required to exhibit the relation between the spaces, velocities, and times, by means of the logarithmic curve.

13. Find where the effect of the ablatitious force to accelerate a body's motion in its orbit is a maximum.

14. The distance between the 10 and 11 o'clock hour-line on a vertical east dial is 3 inches in length; required the height of the style.

First Afternoon.—Mr. HORNBUCKLE.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. If 7 horses eat 16 acres of grass in 20 days, how many will be required to eat 24 acres in 7 days?

2. Add together $\frac{3}{4}$ of 5*s*. 6*d*., $\frac{2}{3}$ of $\frac{1}{5}$ of 17*s*. 2$\frac{1}{4}$*d*., and $\frac{5}{7}$ of £5. 6*s*. 7$\frac{1}{4}$*d*.

3. If a solid angle be contained by three plane angles, any two of these are greater than the third.

4. Prove that the sum of the tangents of two arcs is to the difference as the sine of the sum of these arcs is to the sine of the difference.

5. Solve the equation $x^3 - 3x^2 - 4x + 12 = 0$.

6. Find the quantity Q, so that $Q.(A^2 - B^2)$ may be a perfect c^{th} power.

7. Find the center of gyration in a circle which revolves round an axis passing through its center perpendicular to its plane.

8. Find the radius of curvature at any point of the cycloid.

9. Find the time of flight of a body projected at an angle of 30^0 with a velocity of 193 feet in a second, the mark being in a plane inclined at an angle of 60°.

10. Prove that the sun's image through a convex lens is a circle, and find how the density of rays therein varies.

11. Three persons divide a cylindrical pipe of wine between them, which is emptied through a small orifice in the bottom in 3 quarters of an hour,

each taking as his portion what is emptied in 1 quarter of an hour. In what proportion must they pay for it?

12. Explain the principle of the hydrometer, and shew, that if it be made to sink to the same depth in different fluids, the specific gravities of the fluids will be as the weights of the instrument.

13. Find the proportion in which the spaces occupied by the air in the gauge decrease after each turn of the piston in a condenser.

14. In what part of his orbit is the brightness of Mars the least?

15. Shew where a superior planet is a morning or evening star.

16. Find the solar ecliptic limits.

17. Compare the angular velocity of the distance with the angular velocity of the perpendicular, in general, and in the conic sections.

18. Investigate the expression for the difference of the forces, when the moveable orbit is an ellipse with the center of force in the center.

19. How must the force of gravity be altered that the times of oscillation in a circular arc may be equal.

20. Find the fluxions of $(a+bx+cx^2)^{\frac{1}{2}}$, $\frac{1}{\sqrt{(a+x)}}$ and $(a^x)^x$.

21. Find the following fluents $\frac{2a\dot{x}}{x\sqrt{(a^2+x^2)}}$, and $\frac{rx^{\frac{1}{2}}\dot{x}}{\sqrt{(r^3-x^3)}}$.

22. Determine the apparent path of a star's aberration parallel to the ecliptic, supposing the earth to describe any one of the conic sections.

First Afternoon.—Mr. HORNBUCKLE.

FOR THE FIFTH AND SIXTH CLASSES.

1. Find the chord of curvature, at any point of a parabola, parallel to its axis.

2. Find the altitude, horizontal range, and time, supposing a body to be projected at a given angle with a given velocity.

3. Prove that the density of the air is proportional to its compressing force

4. Prove the rule for division of fractions.

5. Find the equation of limits to the general equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$.

6. Find the law of force tending to the center of the logarithmic spiral.

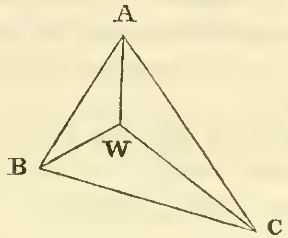
First Evening.—Mr. SOWERBY.

1. It is required to find all the possible values of any two numbers X and Y, so that the difference of their squares may be equal to the square of 24.

2. Of all the lines drawn through the focus of an ellipse, and terminated both ways by its perimeter, that is the least which is perpendicular to the major axis. Required a demonstration.

3. It is required to determine that arc of a given circle, whose cosine is equal to its tangent.

4. Let a given weight (*W*) be supported by three props, *A*, *B*, *C*, as in the annexed figure. The pressure upon each prop is proportional to the area of the triangle opposite to it; that is, the pressure on *A* : pressure on *B* :: area of the triangle *BWC* : area of the triangle *AWC*.



5. Given the length of a cylindrical beam, having its ends placed upon two planes inclined to the horizon at given angles. It is required to find the position in which the beam will rest.

6. A lever whose arms are inclined to each other at a given angle, and whose lengths and weights are respectively known, is made to vibrate flatways round an axis of suspension which passes through the angular point of the lever. It is required to determine the actual time of an oscillation.

7. A cone of given dimensions is filled with fluid, and placed with its slant side parallel to the horizon. How long will the fluid be in running out of it, through a given orifice in the vertex?

8. The velocity and direction of the wind being known, and also the velocity and direction of a ship in motion; it is required to find the position of the

sail with respect to the wind, so that the ship may be impelled with the greatest force possible.

9. Given the difference of the times of setting of two stars whose declinations are known. It is required to determine the latitude of the place.

10. Supposing the latitude and longitude of a star to be known; it is required to determine at what hour it will pass the meridian on that day, when its apparent latitude is neither increased nor diminished by the aberration.

11. In the latitude of 52°, the substyle-line of a vertical dial coincides exactly with the hour-line of 11 o'clock. What is the position of the plane of the dial?

12. A cylindrical vessel of given dimensions is filled with water, and placed with its side perpendicular to the horizon. At what distance from the vessel must a person stand, so that he may just see the center of the base of the cylinder; the ratio of the sines of incidence and refraction being given; and also the height of the eye above the surface of the fluid?

13. Supposing that the periodic times of two bodies revolving in a given circle are the same, and that the one is acted upon by a force situated in the center, the other by a force situated in the circumference. What is the relation of the absolute forces?

14. What must be the law of the force acting upon a body in a logarithmic spiral, so as to cause it to descend from any point in the curve to the center, always in the same time?

15. Supposing a repulsive force to vary inversely as the cube of the distance from a given plane; it is required to determine the trajectory described by a body projected with a given velocity, and at a given distance from the plane, in the direction of a line parallel to the plane.

16. Supposing the earth and moon to move in circular orbits, and that the radii of their orbits and periodic times are known; it is required to determine whether the moon's orbit in fixt space, is concave or convex to the sun, when the moon is in conjunction.

17. It is required to determine the law of the resistance (according to the method of NEWTON, vol. II. Sect. 4.) by which a body may be made to revolve in a parabola round a center of force situated in its focus, the force varying as any power of the distance.

18. In the equation $\frac{p\dot{x}}{x} + \frac{r\dot{y}}{y} = \frac{x^m\dot{x}}{ay^n}$; it is required to find the relation between x and y, when p and r are any numbers whatever.

19. Find the following fluents:

I. $\frac{\dot{x}}{x} \cdot (a^2 + x^2)^{\frac{5}{2}}$.

II. $V^2\dot{x}$, where $V =$ hyp. log. $(1 + x)$.

III. $Xx^2\dot{x}$, where X is a circular arc whose radius $= 1$, and tangent $= \sqrt{\frac{x}{r}}$.

20. Sum the following series:

I. $\frac{1}{1.5} - \frac{1}{2.6} + \frac{1}{3.7} -$ &c. to n terms.

20. Given the sum of the series

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ \&c.$$

to find the sum of the series

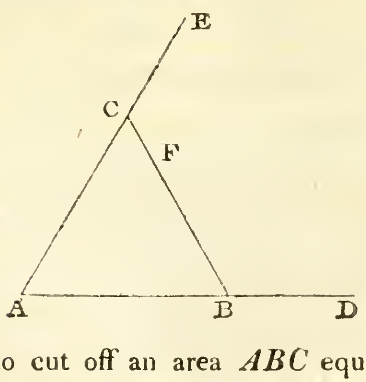
$$\frac{1}{1\,.\,2^2\,.\,3^2}+\frac{1}{2\,.\,3^2\,.\,4^2}+\frac{1}{3\,.\,4^2\,.\,5^2}\ \&c.$$

21. Required the nature of the curve along which a heavy body, descending by the force of gravity, will press upon the curve at any point, with a force proportional to the ordinate at that point.

22. It is required to determine that point in a parabola, to which a line drawn from the vertex makes the greatest angle with the curve.

23. A ship sails from the equator on a north west course. What number of miles will she have run when arrived at the pole, and what will be her difference of longitude?

24. Let *AD, AE* be two lines given in position; and let the line *BC* be moved between them so as



always to cut off an area *ABC* equal to a given area. It is required to find the nature of the curve generated by a point F, which divides the line BC in any given ratio.

25. Let the roots of the equation $x - px^{n-1} + qx^{n-2} - \&c. = 0$, be a, b, c, &c. and those of the equation $nx^{n-1} - (n-1).px^{n-2} + (n-2).qx^{n-3} - \&c. = 0$, be α, β, γ, &c.; then, if when α, β, γ, &c. are successively substituted in the equation $x^n - px^{n-1} + qx^{n-2}$ &c. $= 0$, the results are P, Q, R, &c. and when a, b, c, &c. are substituted in the equation $nx^{n-1} - (n-1).px^{n-2} - (n-2).qx^{n-3} - \&c. = 0$, the results are p, q, r, &c. it will be as $P \times Q \times R$, &c. : $p \times q \times r$, &c. :: 1 : n^n.

Second Morning.—Mr. Sowerby.

Tuesday, January 15, 1805.

1. If a series of arcs be taken in arithmetic progression, the radius of the circle will be to twice the cosine of their common difference, as the sine of any one arc taken as a mean, to the sum of the sines of any two equidistant extremes. Prove this proposition; and shew how, by means of this property, a table of sines, tangents, &c. may be constructed.

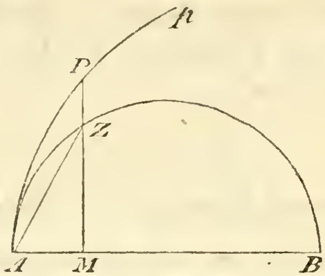
2. Find the sum of n terms of the series

$$\frac{5.6}{1.2.3.4} + \frac{6.7}{2.3.4.5} + \frac{7.8}{3.4.5.6} + \&c.$$

3. Suppose a person of given weight to be suspended in a scale from the extremity of an immoveable lever, and to press upwards by means of a rod

of a given length, against the under-side of the lever; with what force must he press upwards, so that he may rest in any given position?

4. The curve *APp* is generated by taking its ordinate *MP* always equal to the corresponding chord of the circular arc *AZ*. Required its nature, and also its area, supposing it to terminate when its abscissa *AM* becomes equal to the diameter *AB* of the circle.



5. If a sphere and cylinder of the same diameter move with equal velocities in the same fluid, in the direction of the cylinder's axis, the resistance opposed to the motion of the globe will be to the resistance opposed to the cylinder in the ratio of one to two. Required a demonstration.

6. Given the greatest and least horizontal parallaxes of the moon. It is required to find her mean distance in terms of the radius of the earth.

7. To determine the *nature* and *length* of the caustic, when the reflecting curve is a circular arc, and the focus of incident rays is in the circumference of the circle.

8. A body is projected, at a given distance from a

centre of force, with a velocity and direction which will cause it to move in the reciprocal spiral, the force varying inversely as the cube of the distance. It is required to investigate Cotes's construction for determining the place of the body at the end of any given time.

9. Given the densities of the Earth and Jupiter, the times of their diurnal revolutions, and the polar and equatorial diameters of the Earth, to find the ratio between the polar and equatorial diameters of Jupiter.

10. Explain the CARTESIAN hypothesis of vortices, and shew that it will not satisfactorily account for the phænomena of the heavenly bodies.

11. Given the latitudes and longitudes of two places upon the surface of the earth. It is required to determine their distance from each other upon MERCATOR's chart.

12. Supposing the sections of a groin, made by a plane passing through its axis, and cutting the opposite sides of the base at right angles, to be circles; what will be the nature of the section when the plane cuts the sides of the base at any other angle?

13. Given the latitude of the place, the declination of the sun, and the position of a plane, both with respect to the meridian, and the horizon. It is required to find at what hour of the day the sun will begin to shine upon it.

14. If the cover of a common lamp in the streets be a perfect circle, whose plane is perpendicular to

the wall to which the lamp is attached; what will be the nature of its shadow on the wall, supposing the wick to be a point situate in the axis of the cover?

Second Afternoon.—Mr. Sowerby.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the vulgar fraction which is equivalent to the circulating decimal ,7485353.

2. In the extraction of the square roots of numbers, it is required to investigate the limit which the remainder after any operation can never exceed.

3. Find the roots of the equation $x^3 + \frac{10}{7}x^2 - \frac{4000}{9261} = 0$, two of them being equal.

4. A weight (P) draws another weight (W) along an inclined plane given in position, by means of a rope passing over a fixt pully. It is required to find the position of the weight when in equilibrio.

5. In what direction must a ray issuing from a given point be incident upon a given plane refracting surface, so that after refraction it may converge to another given point in the axis of the pencil.

6. In determining the velocity of a fluid issuing from an orifice in the bottom of a vessel, some writers have found it to be that which a heavy body would acquire in falling through the whole height of the fluid in the vessel; others, that acquired in

falling through half the height. How have these different conclusions been obtained, and which is the true one?

7. Suppose a person suspended in a balance to act upwards by means of a rod against a point in the arm of the lever opposite to that in which he is suspended: will his weight be increased or diminished by this action, and in what ratio?

8. Supposing that in the GREGORIAN telescope the focal lengths of the reflectors are given, and also the distance of the last image from the principal focus of the large reflector. It is required to determine the distance of the reflectors from each other.

9. Explain the phænomenon of the harvest moon.

10. Given the difference of the lengths of the shadow of a lofty tower observed at mid-day, in the summer and winter solstices, and also the latitude of the place. It is required to determine the height of the tower.

11. In any conic section, the centripetal is to the centrifugal force as the distance of the body from the focus to half the principal latus rectum of the figure.

12. How far must a body fall *internally*, and how far *externally*, to acquire the velocity which is necessary to retain it in a circle, supposing it in both cases to be acted upon by a force which varies inversely as the square of the distance?

13. Supposing the force in any orbit nearly circular to be represented at any point by the quantity

$\frac{bA^{m} + cA^{n}}{A^3}$, it is required to determine the angle between the apsides.

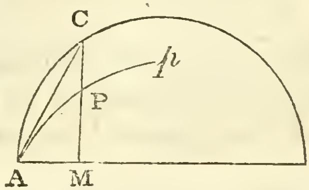
14. Find the sum of n terms of the series,

$$1.2 + 2.3 + 3.4 + 4.5 + \&c.$$

15. To place a given straight line in a circle of given magnitude, so that it may pass through a given point, either within or without the circle.

16. Find the fluent of $\frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}$, and also that of $\frac{\dot{z}}{1+2az+z^2}$, where a is less than unity.

17. Let the curve APp be generated by taking the ordinate MP always equal to the difference of the chord AC, and the versed sine AM of the circular arc AC. It is required to determine the greatest ordinate of the curve, and also its area.



18. If any momenta be communicated to the parts of a system, its center of gravity will move in the same manner that a body equal to the sum of the bodies in the system would move, were it placed in that center, and the same momenta communicated to it, in the same directions.

19. Let the equation to a curve be $xy^2 = a^3$,

where x is the abscissa and y the ordinate. It is required to determine the area comprehended between any two ordinates, b and c.

20. The resistance opposed to a plane moving perpendicularly in a fluid with a given velocity, is to the resistance opposed to its motion when moving obliquely, as the cube of the radius to the cube of the sine of the inclination of the plane to the direction of its motion.

Second Afternoon.—Mr. Sowerby.

1. Investigate a rule for extracting the square root of a binomial, one or both of whose factors are quadratic surds; and apply it to determine the square root of $2\sqrt{-1}$.

2. The spaces described in any times by bodies uniformly accelerated, are proportional to the squares of the times, or to the squares of the last acquired velocities.

3. Explain the principle and construction of the air-pump; and compare the density of the air in the receiver at first, with its density after any number of turns.

4. Find the principal focus of a sphere, the radius of the sphere and the ratio of the sines of incidence and refraction being given.

5. Suppose the body to revolve in an ellipse, it is required to find the law of the force tending to the focus.

6. Divide a given angle into two other angles, so that their sines may be to each other in a given ratio; and shew how the value of these angles may be calculated.

Second Evening.—Mr. Hornbuckle.

1. In a given latitude, a pendulum will oscillate once in one second, supposing the earth not to revolve round its axis. Required the angular motion round the axis, that the pendulum may oscillate once in two seconds.

2. $a^x . b^y . c^z$ is a minimum, and $\overline{x+1} . \overline{y+1} . \overline{z+1} = Q$ a constant quantity. Required the relation between x, y, z.

3. Resolve $\dfrac{1}{1 - ez + fz^2 - gz^3 + \&c.}$ into a series of fractions whose denominators are binomials.

4. Determine the apparent magnitude of a straight rectilinear object, placed at a given depth, parallel to a surface of water; the eye being situated at any point in the plane passing through the object perpendicular to the surface.

5. Two perpendiculars of given lengths are situated at a given distance from each other in a horizontal plane. Determine geometrically that point on the plane between them, to which, if lines be drawn from the extremity of each perpendicular, the times of falling down the two inclined planes may be equal.

6. Find the center of gravity of a cylindrical portion of the atmosphere measured from the earth's surface, the force of gravity being constant.

7. If the ordinate (y) of a curve be composed of powers of the abscissa x and constant quantities; having given the increment of x, find the contemporary increment of y.

8. A string of given length is suspended to two tacks any where situated, the length of the string being greater than the distance between them. It is required to find the position of a given weight (w), which slides freely on the string, when at rest.

9. Construct the spiral in which the areas are the measures of the ratios between the ordinates which terminate them.

10. The angle between the apsides in an orbit very nearly circular : $180^0 :: \sqrt{b+c} . \sqrt{mb+nc}$. Prove that the law of the force hence deduced by Newton's method coincides with $\frac{bA^m + cA^n}{A^3}$.

11. Required the time in which a given cylindrical wheel will roll from the top of a given conical hill to the bottom.

12. Required the present value of £1. to be paid at the end of n years, if either of the individuals A or B, whose ages are given, be alive at that time.

13. It is required to find the altitude of the first point of Aries at a given hour, day, and place; the angle also, and point in which the ecliptic cuts the horizon at that time.

14. Given the length and weight of an elastic string, and the force which stretches it, to find the number of vibrations in a second.

15. Construct the fluent $\frac{\dot{z}}{z + 2az^2 + z^3}$, a being less than unity.

16. A reservoir being supplied with water at a given rate, determine the height to which a sluice must be drawn, that the reservoir may be always kept just full; the dimensions of the sluice, and the depth of its base from the surface, being given.

17. Explain the reason why at spring-tides in summer, in north latitudes, the afternoon tide is greater than the morning tide.

18. A particle is attracted towards a straight line given in position and magnitude, the law of the force being the inverse square of the distance. Determine the direction in which the particle will begin to move towards the line.

19. Find the fluents of the following quantities $\dot{x}\int\dot{x}\int\dot{x}\int\dot{x}$; $x^a a^x \dot{x}$ (a being a given quantity);

$$\frac{\dot{x}}{(x-a)^2 . (x-b)^3}.$$

20. Find the integral of the increment $\frac{1}{V V_2 V_5}$, V being one.

21. The roots of the quadratic equation $x^2 - px + 1 = 0$ are a and b. Prove that $a^n + b^n$ is equal to

$$p^n - np^{n-2} + n . \frac{n-3}{2} p^{n-4} - n . \frac{n-4}{2} . \frac{n-5}{3} p^{n-6}$$

+ &c.

22. Bradley observed, that the apparent motion

in declination of every star tended the same way when they passed the meridian at the same hour, each being farthest north when it passed at six o'clock in the evening, and farthest south at six in the morning. Explain the reason why this is nearly true in those stars whose declinations are not very great.

23. A ball is shot from a cannon with a given velocity, at a given angle of elevation, situated at a given distance from the foot of a hill, whose elevation is also known. Determine the point in which the ball will strike the hill.

24. Given the force of gravity at the surface of a primary planet, the mean distance and periodic time of its secondary with the ratio of their respective diameters. It is required to compare their densities.

1806.

First Morning.—Mr. J. Brown.

Monday, January 13, 1806.

1. How many years' purchase is an estate worth, when the rate of interest is 4 per cent.?

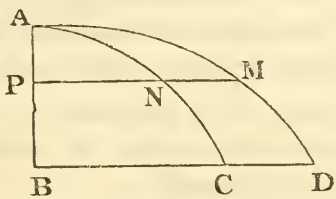
2. An inclined plane is a tangent to a cycloid in the middle point between the highest and lowest points; what must be the length of the plane, that the time of falling down it may be equal to the time of half an oscillation?

3. Having given the specific gravities of water and iron, it is required to determine what proportion the thickness of an hollow iron globe must bear to its diameter, that it may just float in water.

4. In a given latitude, and at a given time of the year, how many hours will be shewn upon a vertical south dial? And at what time of the year will the greatest number be shewn?

5. Having given the altitudes of the sun, and of any particular colour in the primary rainbow, it is required to determine the sines of incidence and refraction.

6. The curve ANC is a cycloid, and the curve AMD is formed by taking PM equal to the arc AN; find the equation to the curve, and compare its area with that of the cycloid.



7. Compare the attractions of a spheroid upon a particle on its surface in directions parallel to the polar and equatorial diameters, with the attractions at the pole and the equator.

8. A paraboloid with its vertex downwards is emptied by an orifice in its vertex; compare the times in which the fluid descends through the first and last half of the axis.

9. Draw a diameter to the curve, whose equation is $y^n - \overline{a+bx} \cdot y^{n-1} + \&c. = 0$.

10. Sum the following series:

$$1^3 + 2^3 + 3^3 + \&c. \text{ to } n \text{ terms.}$$

$$\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} \;\&c. \text{ in inf.}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c. \text{ in inf.}$$

11. Find the following fluents:

$$\frac{x\dot{x}}{\sqrt{(2ax-x^2)}}, \quad \frac{\dot{x}}{1-x^4}, \quad \frac{z\dot{z}}{1+2az+z^2},$$

where a is less than unity; and construct the latter.

12. Trace the curve whose equation is $a^2 - x^2 + (x-b)^2 = x^2 y^2$.

13. Shew that the altitude of high water above the mean altitude is equal to twice the depression below it.

14. Find the radius of curvature of the common parabola, and the equation to the evolute.

15. Shew that if the resistance in any medium is proportional to the velocity, the oscillations in a cycloid will be isochronous.

16. Supposing the moon to revolve round the earth in a circle, what must be the diminution of the quantity of matter in the earth, that the eccentricity of her orbit may be equal to half the radius of her present orbit?

First Afternoon.—Mr. J. Brown.

FOR THE THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the value of .78545454 &c. in a vulgar fraction.

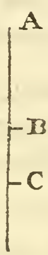
2. If £40. be due at the end of 6 monthss, £60. at the end of 12 months, and £80. at the end of 15 months; what is the equated time for paying the whole?

3. The sum of four numbers in arithmetical progression is 56, and the sum of their squares is 864; What are the numbers?

4. If the latitude of the place, and the sun's

declination be given; it is required to find the time of his rising, his azimuth at that time, and the time of his coming to the prime vertical.

5. A body has fallen from A to B, when another body is let fall from C; How far will the latter body fall before it is overtaken by the former.



6. If R and r be the radii of the surfaces of a convex lens, and m and n the sines of incidence and refraction; find the focal length of the lens, and shew that it is the same which ever way the rays pass.

7. Having given the three angles of a spherical triangle, and the radius of the sphere, find the area of the triangle.

8. Find the ecliptic limits of the sun and moon, and how many eclipses there may be in one year.

9. Inscribe the greatest cone in a given sphere.

10. Find how far a body must fall internally to acquire the velocity in a circle, when the force varies inversely as the square of the distance.

11. When a body revolves in any curve, compare the angular velocity of the distance with that of a perpendicular upon the tangent.

12. Shew that the periodic times in all ellipses round the same center are equal.

13. If the force varies inversely as the cube of the distance, and the whole distance be made the radius of a circle; then, if the versed sine of any angle represent the space fallen through, the right sine will represent the time, and the tangent the velocity acquired. Required a proof.

14. Solve the following cubic equation, whose roots are in geometric progression;

$$x^3 - 13x^2 + 39x - 27 = 0.$$

15. Shew that the time of an oscillation in a cycloid : time down the axis :: the circumference of a circle : the diameter. And having given the length of a pendulum, which vibrates seconds, find how far a body will fall by the force of gravity in one second.

16. Compare the times of emptying a cone and its circumscribing cylinder, through equal orifices in the base of the cylinder, and the vertex of the cone.

17. Shew that if no light were intercepted in passing through the air, an object would appear equally bright at all distances.

18. A barometrical tube is 36 inches long, and after inversion the mercury stands at the height of 24 inches. What quantity of air was left in the tube before inversion, supposing the standard height to be 30 inches?

19. Find the following fluents:

$$\frac{2a\dot{x}}{a^2-x^2}, \quad \frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}, \text{ and } \frac{\dot{z}}{1+2az+z^2},$$

where a is less than unity.

20. Sum the following series:

$$\frac{3}{1.2.2}+\frac{4}{2.3.2^2}+\frac{5}{3.4.2^3}+ \text{\&c. in inf.}$$

$$1.2+2.3+3.4+4.5 \text{ \&c. to } n \text{ terms.}$$

21. If a, b, c, &c. be the roots of the equation $x^n-px^{n-1}+qx^{n-2}-$ &c. $=0$; it is required to transform it into an equation, whose roots are ma, mb, mc, &c.

22. Shew that the attractions of similar pyramids, whether whole or obtruncated, upon particles situated in their vertices, are proportional to their lengths.

23. At what angle must a body be projected with a given velocity, that the area of the parabola described may be a maximum?

24. Determine the angle between the apsides when the force varies as $\frac{bA^m+cA^n}{A^3}$.

First Evening.—Mr. Barnes.

1. The sum of the cosines of any two arcs, is to their difference, as the co-tangent of half their sum, to the tangent of half their difference.

2. Given the latitude of the place, and the right ascension and declination of a star, to find the hour at which it will rise on a given day.

3. Find the focal length of a thin concavo-convex lens of glass, the radii of the surfaces being given.

4. Construct and investigate the magnifying power of the double microscope.

5. The roots of the equation $4x^3 - 32x^2 - x + 8 = 0$ are of the form $a, -a, b$. It is required to determine them.

6. If a be an integral root of the equation $x^n + px^{n-1} + \&c. \ldots + Px^3 + Qx^2 + Rx + S = 0$, and if $\frac{S}{a} + R = R'$; $\frac{R'}{a} + Q = Q'$; $\frac{Q'}{a} + P = P'$ &c.; then $\frac{S}{a}$, R', Q', P' &c. are integers. Required a demonstration.

7. Given the specific gravities of two bodies, to find the ratio of their quantities of matter, when they balance each other in a fluid whose specific gravity is also given.

8. A given cylindric vessel is supplied with water by a cock at a given rate. Then suppose when the vessel is full, that an aperture of given area is made in its bottom; what is the lowest point to which the surface of the water in the vessel will descend, and also the time of its descent; the influx of the water by the cock being supposed slower than the efflux when the vessel is full?

9. A given weight, suspended by a thread of a given length, oscillates, in the manner of a pendulum, through a semi-circle. Compare the weight of the body with the force which stretches the string in any position.

10. Find the magnitude of that part of the disturbing force of the sun upon the moon, which acts in a direction perpendicular to the plane of the moon's orbit.

11. Required the fluents of $x^3\dot{x} \times (a^2+x^2)^{\frac{3}{2}}$, $\dfrac{d \times z^{\frac{1}{2}n-1}\dot{z}}{(a+bz^n)\sqrt{(c+ez)^n}}$, and $\dfrac{\dot{z}}{z^n \times (a+bz)}$.

12. Find the relation of x to y in the equation $x\dot{y} - y\dot{x} = \dot{x}\sqrt{(x^2+y^2)}$.

13. Required the sum of n terms of the series $1.2.4+2.3.5+3.4.6+$ &c.; and the sum of the series $1+\frac{1}{3.5}+\frac{1}{5.9}+\frac{1}{7.13}+$ &c. ad inf.

14. Given the sum of the series, $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ &c. equal to $\frac{2Q^2}{3}$, (Q being the length of the quadrantal arc to the radius $= 1$); to find the sum of the series, $\frac{1}{1^2.2}+\frac{1}{2^2.3}+\frac{1}{3^2.4}+$ &c. ad inf.

15. The fluent of $y\dot{x} = y \times \frac{x}{1} - \frac{\dot{y}}{\dot{x}} \times \frac{x^2}{1.2} + \frac{\ddot{y}}{\dot{x}^2} \times \frac{x^3}{1.2.3} -$ &c. ($\dot{x}$ being constant); required the investigation.

16. At what distance from its extremities must a slender rod, of uniform thickness and given weight, be suspended, that it may oscillate in a given time?

17. Suppose the proportion of the chances for the happening of an event, be to that of its failing in any one trial as a to b; what is the probability

that the event will happen precisely p times in n trials?

18. If z be constant,

$$\frac{1}{z} = \frac{1}{z_n} + \frac{nz}{z_n z_{n+1}} + \frac{n(n+1)z^2}{z_n z_{n+1} z_{n+2}} + \&c.$$

required the investigation.

19. If a body revolve in an ellipse round a center of force situated in the focus; investigate the relation between the mean angular velocity of the body, and its angular velocity round the upper focus; and shew from this relation that when the ellipse is of small excentricity, the angular velocity round the upper focus is nearly uniform.

20. The greatest velocity which can be acquired by a spherical body descending in a fluid, is equal to that which would be acquired by it in descending from rest in vacuo, by the constant force of its comparative gravity, through a space which is to $\frac{4}{3}$ of the diameter, as the density of the sphere to the density of the fluid.

21. Suppose the force of gravity to be uniform, and to tend perpendicular to the horizon; what will be the path of a projectile in a medium, the resistance of which is proportional to the velocity of the body?

22. If the centripetal force vary reciprocally as the distance from the centre; and the density of the fluid be proportional to the compressing force; then if distances from the centre be taken in geometrical progression, the corresponding densities of the fluid will be also in geometrical progression.

23. Bradley observed that every star passed the meridian farthest south when it came about six in the morning, whatever were its position with respect to the cardinal points of the ecliptic. Suppose then the place of a star to be given; on what day will it pass the meridian of London farthest to the south; and at what hour will it pass it on that day?

24. A perfectly flexible chain is wound round a cylinder supported with its axis parallel to the horizon. Then, if the weight and dimensions of the cylinder be given, and also the length and weight of the chain; it is required to determine the time in which the chain, impelled by the force of gravity, will unwind itself; a given length being unwound at the commencement of the motion.

Second Morning.—Mr. Barnes.

1. Find the value of the decimal ,726363 &c. in a vulgar fraction.

2. Required the value of x in the equation $2x\sqrt[3]{x} - 3x\sqrt[3]{\frac{1}{x}} = 20$.

3. If $x^n - px^{n-1} + qx^{n-2} -$ &c. $\pm W = A$, and a be any root of the equation $x^n - px^{n-1} + qx^{n-2} -$&c. $\pm W = 0$; Prove that $x - a$ is a divisor of the expression $x^n - px^{n-1} + qx^{n-2} -$&c. $\pm W$.

4. Suppose the arms of a bent lever, inclined to each other at a given angle at the fulcrum, to be also given: Required the position of the lever, when

two given weights suspended freely from its extremities, balance each other.

5. A given orifice is opened in the bottom of a vessel of given altitude filled with water; what must be the nature and dimensions of the vessel, that the surface of the water may descend with a given uniform velocity?

6. Required the nature of the caustic, when the reflecting curve is the common cycloid, and the rays are incident parallel to the axis.

7. Two stars whose right ascensions and declinations are known, were observed to rise at the same moment. Required the latitude of the place of observation.

8. Prove that the periodic times of bodies describing different ellipses round different centres of force situated in their foci, are in the sesquiplicate ratio of the major axes directly, and in the subduplicate ratio of the absolute forces inversely.

9. Find the fluents of

$$\frac{x^3\dot{x}}{\sqrt{(a^2-x^2)}} \text{ and of } \frac{\dot{x}}{x^3\sqrt{(a^2-x^2)}}.$$

10. Required the sum of the first n terms of the series $\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}$ &c. n being an even number.

11. Find the sum of the first n terms of the series $\frac{1}{1.2.4}+\frac{1}{2.3.5}+\frac{1}{3.4.6}+$ &c. by the method of increments.

12. Compare the mean quantity of the force by

which the sun disturbs the motion of the moon, with the force of gravity at the earth's surface.

13. Compare the force which accelerates the centre of gravity of a cylinder, while it rolls down a given inclined plane, with the weight of the cylinder.

14. Given the latitude of the place, and the right ascension and declination of a star, to determine the time of the year when the star rises with the sun.

Second Afternoon.—Mr. Barnes.

FOR THE THIRD, FOURTH, FIFTH AND SIXTH CLASSES.

1. What is the interest of £400. for 9 months at $4\frac{3}{4}$ per cent. per annum?

2. What is the discount of the same sum for the same time, at the same rate of interest?

3. Reduce 1.6363 &c. of a crown to the fraction of a guinea.

4. Given $3x^2 - 3x + 6 = 5\frac{1}{3}$, to find x.

5. Required the sum of n terms of the series, $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} -$ &c.; and also the sum of the series continued ad infinitum.

6. Find the sum of n terms of the series a, $a+b$, $a+2b$, $a+3b$, &c.

7. With what velocity must a body be projected from a given point in a given direction, to hit a given mark?

8. Given the specific gravity of a fluid, and also of a solid in the form of a paraboloid floating in it with its vertex downwards, to find the part of the axis immersed.

9. Given the distance of the object from a double convex lens, and the ratio of the object to the image, to find the focal length of the lens.

10. If a body revolve in an ellipse round a center of force situated in one of its foci; prove that the velocity in any point, is to that in a circle at the same distance, in the sub-duplicate ratio of the distance from the other focus, to half the major axis.

11. Given the latitude of the place, and the declination of the sun, to find the length of the day.

12. If the signs of the alternate terms of an equation beginning with the second, be changed, the signs of all the roots are changed.

13. Required the fluxions of

$$\frac{a}{\sqrt{(x^2+y)}}, \text{ and of } \frac{(x+a)^2}{\sqrt{(x^2-a^2)}}.$$

14. Find the fluents of

$$\frac{bx\dot{x}}{\sqrt{(a+x)}}, \text{ and of } \frac{2ax^2\dot{x}}{a^2-x^2}.$$

15. If the force vary directly as the distance from the center, shew that the velocity of a falling body, is to that in a circle at the same distance, as the right sine to the cosine of a circular arc, whose radius is the greatest distance, and versed sine the space through which the body has fallen.

16. At what season of the year is that part of the equation of time, which arises from the inclination of the ecliptic to the equinoctial, a maximum?

17. If two imperfectly elastic balls strike each other, prove that the relative velocity before impact, is to the relative velocity after, as perfect to imperfect elasticity.

18. If perpendiculars be drawn from any number of bodies to a given plane, the sum of the products of each body multiplied by its perpendicular distance from the plane, is equal to the product of all the bodies multiplied by the perpendicular distance of their common center of gravity from the plane.

19. Investigate the fluxional expression for the area of any curve; and apply it to find the area of the curve whose equation is $a^m x^n = y^{n+m}$ comprehended between the ordinates, whose lengths are b and c.

20. Find that point in the moon's orbit (supposing it a circle,) where the disturbing force of the sun, neither increases nor diminishes the gravitation of the moon towards the earth.

Second Afternoon.—Mr. Barnes.

FIFTH AND SIXTH CLASSES.

1. Reduce $\frac{2}{3}$ of a guinea to the decimal of a pound.

2. Prove the rule for finding the greatest common measure of two numbers.

3. Reduce the fraction $\frac{209}{380}$ to its lowest terms.

4. Required the value of x in the equation $2x^4 - x^2 + 104 = 600$.

5. Prove that if a body be projected in any direction inclined to the horizon, it will describe a parabola.

6. Find the law of the force by which a body is made to revolve in a parabola, round a center of force situated in the focus.

7. Shew that the pressure of the fluid upon any surface immersed in it, is equal to the weight of a cylindric column of the fluid, whose base is the surface pressed, and altitude the perpendicular depth of the center of gravity of the surface pressed below the surface of the fluid.

8. Construct and investigate the magnifying power of the common astronomical telescope.

Second Evening.—Mr. J. Brown.

FOR THE FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. A pendulum, which kept true time at the earth's surface, when carried to the top of a mountain lost n seconds a day; What was the height of the mountain?

2. Having given the places of the sun and moon at any time, it is required to delineate the phases of

the moon, and to find the angle which her cusps make with the horizon.

3. Shew in what positions of the eye and object, the image formed by a double convex lens will be magnified or diminished; and when it will be erect, and when inverted.

4. Having given the weight and magnitude of a balloon, and the specific gravities of atmospheric air, and of the air with which the balloon is filled, find the height to which it will ascend.

5. Shew how an eclipse of the moon, with the time of its duration, and the number of digits eclipsed, may be calculated.

6. Compare the times of emptying a globe and its circumscribing cylinder through equal orifices; the orifice of the cylinder being in the base.

7. A given weight P is connected with a cylinder by means of a string wound round it, and descends 48 feet in 2 seconds, causing the cylinder to revolve round its axis. What is the weight of the cylinder?

8. A cylindrical vessel, whose height is equal to its diameter, is filled with water. With what velocity must it be whirled round its axis, that half the water may be thrown out?

9. Approximate to the root of the equation $x^3 + x^2 + x = 90$; and shew that the accuracy of the approximation depends upon the quantity assumed being much nearer to one root than to any other.

10. If the earth and moon were at rest, having given their quantities of matter and densities, it is

required to determine the least velocity, with which a body projected from the moon would fall to the earth.

11. If a body be projected from an apse with the velocity acquired in falling from an infinite distance when the force varies inversely as the n^{th} power of the distance. In how many revolutions, or in what parts of a revolution, will it either fall into the center, or go off to infinity?

12. When a globe oscillates in a resisting medium having given the diameters of the globe and of the string, compare the resistance upon the string with the resistance upon the globe.

13. Divide a given angle into two such parts, that the cube of the sine of one part multiplied into the square of the sine of the other may be a maximum. And give the geometrical construction for dividing the angle.

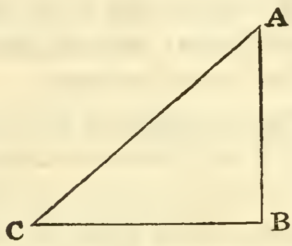
14. Resolve the fluxional equation $x\dot{x} + ay\dot{x} - y\dot{y} = 1$.

15. Supposing the moon's orbit nearly circular, find the increase of her velocity by the tangential force, while she moves from quadrature to syzygy.

16. Resolve $\frac{1}{1+x^n}$ into its quadratic divisors.

17. The height of the tower AB is to be determined by measuring the base BC, and taking the angle ACB; supposing there to be some small error in the observation, at what distance from the foot of the tower should it be made, that the corre-

sponding error in the height of the tower may be a minimum?



18. Having given the fluent of $(a+cz^n)^m \times z^{pn-1}\dot{z}$, find the fluent of $(a+cz^n)^{m+r} \times z^{pn+vn-1}\dot{z}$.

Find the fluent of $v^2\dot{x}$, where $v =$ the hyp. log. of $\{x+\sqrt{(a^2+x^2.)}\}$

Find the fluent of $\frac{\dot{x}}{\sqrt{(1+a^x)}}$.

19. Find the magnifying power, and field of view, of Cassegrain's telescope.

20. Sum the following series;

$$\frac{1}{1.5}+\frac{1}{3.7}+\frac{1}{5.9} \text{ to } n \text{ terms, and ad inf.}$$

$$\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\frac{1}{7.8} \text{ \&c. ad inf.}$$

$$\frac{1}{1.2.5}+\frac{1}{2.3.7}+\frac{1}{3.4.9} - \text{\&c. ad inf.}$$

$$1-\frac{1}{3.3}+\frac{1}{5.3^2}-\frac{1}{7.3^3}+\text{\&c. ad inf.}$$

21. Having given the diameter of a primary planet, and the force of gravity upon its surface, and also the periodic time and distance of the secondary revolving in a circle, to find the quantity of matter in the secondary.

22. Explain the principle of interpolations.—And having given on the 19^{th} of any month, at noon, the sun's declination = 28′.41″ North; on the 20^{th} = 5′.0″ North; and on the 21^{st} = 18′.41″ South; find the time of equinox.

23 Shew that if a globe of fluid matter revolve round its axis, it will put on the form of a spheroid.

24. Shew that every section of a solid formed by the revolution of a conic section round its principal axis is also a conic section.—And if a paraboloid be cut by a plane parallel to its axis, the section will be a parabola similar and equal to the original one.

25. Shew that the area of the least polygon circumscribing a circle : the area of the greatest polygon of the same number of sides, inscribed in the circle :: $(\text{radius})^2$: $(\text{cos.})^2$ of half the angle subtended by any side.

Mr. J. Brown.

FOR THE FIFTH AND SIXTH CLASSES.

1. The rectangle contained by the diagonals of a quadrilateral figure inscribed in a circle, is equal to both the rectangles contained by its opposite sides.

2. In the collision of two perfectly elastic bodies, the sum of the products of each body multiplied into the square of its velocity, is the same before and after impact.

3 Shew that, when bodies revolve in circles having the force in centre, the force varies as the

radius directly, and the square of the periodic time inversely.

4. Given the sines and cosines of two angles, to find the sines and cosines of their sum and difference.

5. Shew that an arithmetic mean is greater than a geometric one.

6. Sum the following series:

$$12 + 4 + \frac{4}{3} + \frac{4}{9} \text{ \&c. in inf.}$$

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} \text{ \&c. in inf.}$$

7. Explain the cause of the change of seasons, and of the difference of the lengths of day and night.

1807.

First Morning.—Mr. D'Oyly.

Monday, January 19, 1807.

1. Find the least whole number, which, when divided by 3 and 5, has its respective remainders 1 and 3.

2. A spherical triangle has its angles 160^0, 150^0, and 140^0. Compare its area with that of a great circle of the same sphere.

3. The equation $x^3+8x^2+20x+16=0$ has two equal roots. Find the three roots of the equation.

4. The sun's right ascension is 40^0, and the latitude of the place 52^0. Find the ascensional difference.

5. If the force varies as the cube of the distance, how far must a body fall internally to acquire one-third of the velocity in a circle?

6. A lighter fluid rests on a heavier: their specific gravities are respectively 3 and 6. A paraboloid whose equation is $ax=y^4$, floats between them with its vertex downwards, and sinks with one-fourth of its axis in the heavier. Find the specific gravity of the solid.

7. A groin is generated by a square moving parallel to itself, the section through its opposite sides being a semi-cubical parabola. Find the time in which it empties itself through a given hole in the vertex.

8. The periphery of a circle revolves on a diameter for its axis. At what distance from this axis is its centre of gyration?

9. Two telescopes, of Sir I NEWTON's construction, with equal reflectors, have the same degree of indistinctness. Compare their magnifying powers, and their degrees of brightness.

10. Find the relation of x to y in the equation $a\dot{x}^2 = y\dot{y}^2 - b\dot{x}\dot{y}$.

11. There are 4 individuals, of such an age, that, according to the tables of mortality, it is just an even chance whether each will be alive at the end of a year. What is the probability that two of them will die in that time?

12. Sum the series

$$\frac{1}{3} + \frac{1.2}{3.4} + \frac{1.2.3}{3.4.5} + \frac{1.2.3.4}{3.4.5.6} + \text{\&c. ad infin.}$$

also, $\frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9} +$ &c. to n terms.

13. Find fluents of

$$\frac{x\dot{x}}{(x-a)^2}, \quad \frac{x\dot{x}\sqrt{(1-x^2)}}{\sqrt{(1+x^2)}}, \text{ and } \frac{\dot{x}}{\sqrt{(1-e^{-mx})}},$$

where e is the number whose hyp. log. is 1.

First Afternoon.—Mr. Woodhouse.

THIRD AND FOURTH CLASSES.

1. Find the interest on 12*s*. 4*d*. at $4\frac{1}{2}$ per cent.

2. Given the first term $= 1$, the number of terms $= n$, and the sum $= S$; Find the common difference, the series being arithmetical.

3. Prove that sin. $(60 + A) =$ sin. $(60 - A) +$ sin. A.

4. Prove that

$$\frac{\sin. a + \sin. b}{\cos. a + \cos. b} = \tan. \left(\frac{a+b}{2}\right) \text{ (radius} = 1.)$$

5. Compare the sun's apparent diameter at the extremity of the latus rectum of the earth's orbit, with the perigean apparent diameter.

6. If two bodies are moved at the same time towards each other, from the two extremities of a vertical line (L); one projected upwards with a velocity acquired down $\frac{3L}{2}$, the other let fall from rest; It is required to find the point where they meet.

7. A cubical vessel is filled with water, and into its side a bent tube is inserted, filled with water, and communicating with the water in the vessel; Required the pressure on the top of the vessel, the vertical height of the extremity of the tube above the vessel being m times the height of the vessel.

8. Let the focus of incidence Q be $\frac{1}{4}$ radius from

the vertex of a spherical concave reflector; Find the place of q, the focus of reflected rays.

9. Let a weight P, descending vertically, draw Q up an inclined plane, the elevation of which is 30 degrees. Find the velocity of P, after that a time t is elapsed.

10. In Newt. Prop. VIII. Sect. 2. a semi-circle is described by P; what must be the change of circumstances that an ellipse may be described?

11. Find the fluxion of the hyp. log. $\left(\frac{1-x^{\frac{3}{2}}}{1+x^{\frac{3}{2}}}\right)$, and fluent of $x\dot{x}\sqrt{(1+x)}$.

12. Shew by what method the obliquity of the ecliptic is determined.

13. In elastic bodies, shew that, if A overtake B, the velocity of B after impact is greater than the velocity of A.

14. Given $\begin{cases} \text{Latitude} = l, \\ \text{Sun's altitude} = a, \\ \text{Comp. declination} = p. \end{cases}$

It is required to prove; that, t being time from noon,

$$\text{ver. sin. } t = 2 \cos. \left(\frac{p+l+a}{2}\right) \sin. \left(\frac{p+l+a}{2} - a\right) \times \text{cosec. } p \,.\, \text{sec. } l.$$

15. If a point be kept at rest by any number of forces represented by the sides of a polygon, then any one force is equal and opposite to the result of all the other forces. Required proof.

16. Sum $1 + 3 + 6 + 10 + 15$ &c. to m terms.

17. Let a body be projected from the top of a tower horizontally, with a velocity acquired in fall-

ing down a space equal to the height of the tower. At what distance from the base of the tower will it strike the horizon?

18. Sum $1^3+2^3+3^3+\&c.\ldots\ldots n^3$. by the method of increments.

19. In the equation $x^3-\frac{7}{2}x^2+\frac{7}{2}x-1=0$, two roots are of the form a and $\frac{1}{a}$. It is required to solve the equation.

20. Compare the velocity of a body moving in a parabola, at the vertex, with the velocity at the extremity of the latus rectum; force being in the focus.

21. Let a body revolve in an hyperbola. Required the law of the centrifugal force directed from the center of the hyperbola.

Mr. Woodhouse.

FIFTH AND SIXTH CLASSES.

1. What is the interest of £1. 1*s.* 6*d.* for one year, at 5 per cent.?

2. Find 4 numbers in arithmetical progression, the common difference being 3, and their product 280.

3. Prove, that tan. $2A$ is greater than 2 tan. A, $A \angle 45°$.

4. Let 3 equal bodies be placed in the 3 angles of a triangle. Find their center of gravity.

5. Compare the velocity of a body at the extre-

mity of the latus rectum of an ellipse, with the velocity at the mean distance; force being in the focus.

6. The specific gravities of mercury and water being as 13568 to 1000. What ought to be the length of a water barometer inclined to the horizon at an angle of 30^0, the mercury standing at 30. 5 inches?

7. In north latitude $52^o. 12'. 36''$, and with $1^o. 30'. 0''$, north declination, how long in 24 hours is the sun above the horizon?

8. Find the fluxion of $\frac{x}{1+x^2}$, and the fluent of $\dot{x}(1+ax)^{-\frac{3}{2}}$.

9. Q, the focus of incidence, is distant 100 radii from the vertex of a concave spherical reflector. Find the place of q, the focus of reflected rays.

10. S, S', S'', are the sums of 3 arithmetical series; 1 is the first term in each, and the respective differences are 1, 2, 3; then

$S+S''=2S'$. Required proof.

11. . . . . . . $x^2+y^2=5$,

$3xy \quad =6$.

Find x and y.

Evening Problems.—Mr. Woodhouse.

1. Required the amount on £30. 15*s.* at 3*d.* in the pound.

2. The first term in an arithmetical series is 1, and the difference is m; then the sum of n terms $= \frac{mn^2 - (m-2)n}{2}$. Required proof.

3. Let $x^2 - px - q = 0$, assume tan. $z = \frac{2\sqrt{q}}{p}$, rad. = 1; then the two roots of the quadratic are,

$$\frac{p}{2}\text{tan. } z.\text{cotan. }\frac{z}{2}, \text{ and } -\frac{p}{2}\text{ tan. } z.\text{tan. }\frac{z}{2}.$$

Required proof.

4. Prove, that in every quadrilateral figure, the sum of the squares of the two diagonals is double of the sum of the squares of the two right lines that join the middle points of the opposite sides.

5. If £1. at compound interest, rate r, doubles itself in n years; and, rate $2r$ in m years; what is the ratio between n and m? Is it less or greater than, or equal to, the ratio of 2 to 1?

6. If 2 cos. θ be assumed $= u + \frac{1}{u}$, rad. = 1, then,

$$2\cos. 2\,\theta = u^2 + \frac{1}{u^2},$$

$$2\cos. 3\,\theta = u^3 + \frac{1}{u^3},$$

and generally, $2\cos. n\theta = u^n + \frac{1}{u^n}$. Required proof.

7. Find the fluxion of h. l. $x.e^{\cos. x}$, e being the base of the hyperbolic logarithms.

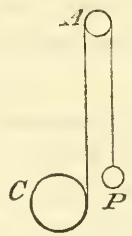
8. Find the fluents of

$$\frac{x\dot{x}}{(1+x)^{\frac{3}{2}}}, \text{ and of } \frac{\dot{x}}{1+a.\cos. x}, \; a \angle 1.$$

9. A pipe, cylindrical within, is placed vertically, filled with a fluid: what ought to be the law of the increase of the thickness of its sides, that it may be equally strong throughout; the increase of strength being supposed to vary with the increase of thickness?

10. Let two bodies, with velocities v, v', be projected at the same time and towards each other, from the two extremities of a vertical line; then they shall always meet in the middle of the line, if the difference of the squares of the velocities v, v', is equal to the square of a velocity acquired in falling down half the line.

11. The weight P is attached to a string which goes over a fixed pulley at A, and is wound round



a thin cylinder C, the weight of which is W. Required P, such, that it shall neither ascend nor descend, whilst C descends by the unwinding of the string.

12. If from the four angles of a pyramid there be drawn lines to its center of gravity, then a point there placed, shall be kept at rest by forces represented by the 4 lines. Required proof.

13. Let the axis of motion pass through a thin

straight rod of uniform thickness and density, dividing the rod into 2 arms that have the ratio of 2 to 1. It is required to find the velocity acquired by the extremity of the longer arm, when the rod shall have moved from an horizontal to a vertical position.

14. If the sine of incidence be to the sine of refraction as 1 : n, then the focal length (f) of a double convex lens, may be nearly had from this expression,

$$\frac{1}{f} = \frac{1-n}{n}\left\{\frac{1}{r} + \frac{1}{r'} + \frac{1-n}{r^2}t\right\},$$

r, r' being the radii of the two surfaces, and t a very small quantity, the thickness of the lens. Required proof.

15. Prove, that the earth's angular velocity at mean distance is less than the mean angular velocity.

16. Prove, that the augmentation of the sun's apparent diameter from apogee is nearly proportional to the versed sine of angular distance from apogee; the excentricity of the earth's orbit being a small quantity.

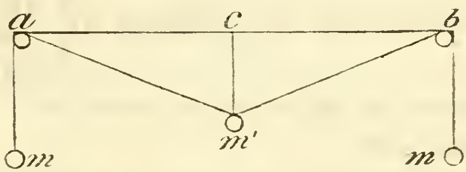
17. Given the right ascension and declination of the moon's center, and of a fixed star. Find their distance.

18. Given the latitude of a place, the sun's declination and altitude. Find the azimuth, and under a form adapted to logarithmic computation.

19. The line of the nodes being in octants, and the body P (Newt. Sect. 11.) in quadratures, com-

pare the disturbing force that acts perpendicularly to the plane of the orbit, with the like force, when the line of the nodes is in quadratures, and the body in octants; the inclination of the orbit being supposed to be the same.

20. a, b, are two fixed pullies over which the string passes, to which m, m', m, three equal



weights, are fastened. Suppose m' to be let fall from c, it is required to find the distance from c, at which its velocity is the greatest.

21. Let a weight P, fastened to a string going over a wheel, by its descent, cause two weights, Q, Q', to be wound up on two axles. Required the velocity of P, after that it has descended through a space s; the radii of the wheel and of the two axles being r, r', r''.

22. A and B play at a game, each with an equal chance of success, and with this condition; A gives to B a sum S if he loses the first time; $2S$, if he loses two games successively; $4S$, if three games successively, &c.; and $2^{n-1}S$, if he loses n games successively. What sum, on the received principles of chances, ought B to give to A, as an equivalent of the risque he encounters?

Tuesday Morning Papers.—Mr. WOODHOUSE.

FIRST AND SECOND CLASSSES.

1. Given, the logarithm of 8.1213 = .9096256;

It is required to state the logarithms of $\left\{\begin{array}{r} 81.213 \\ 812.13 \\ .81213 \\ .081213 \end{array}\right\}$

and to prove the justness of the operation.

2. Prove that

$$\tan.\ (45+A) = \tan.\ (45-A) + 2\tan.\ 2A.$$

3. Prove that the tangential part of the ablatitious force varies as sin. 2θ, θ being the angular distance of P from quadratures (NEWT. Sect. 11.)

4. The force varying as the distance, let a body be projected with a certain velocity, at a given distance *(d)* from the center, at an angle, the sine of which is S. It is required to prove that the orbit described must have an apse.

5. State the usual distinction between apparent solar year, and mean solar year; and explain the cause of their difference.

6. Explain the method of finding the right ascension and declination of a star.

7. Given the right ascension and declination of a star; find its longitude.

8. Prove that $\int \frac{\dot{z}}{(\cos.\ nz)^2} = \frac{1}{n}\tan.\ nz.$

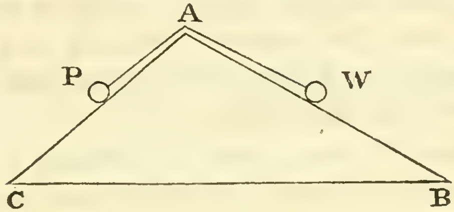
9. If the hyp.log. of n be given, then the hyp.

log. $(n+1)$ may be conveniently found from this formula: hyp. log. $(1+n)$ =

$$\text{hyp. log. } n + 2\left\{\frac{1}{2n+1} + \frac{1}{3}\cdot\frac{1}{(2n+1)^3} + \&c.\right\}$$

Required proof.

10. Given the plane AC; It is required to find the elevation of the plane AB, such, that W shall



be drawn by P from the horizontal line BC to the point A in the least time possible.

11. Find the integral of $\frac{4z+3k}{z.(z+k)(z+2k)}$, k being the increment of z.

12. Let $y = \frac{x}{1+x^2}$; trace the curve, find the values of y when a maximum, and at the point of contrary flexure; and find the value of the angle in which the curve cuts the axis at the point of intersection.

13. Investigate a formula for clearing the moon's distance from a star, from the effects of parallax and refraction.

Morning Problems.—Mr. D'Oyly.

THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. What is the discount on £80. 10*s*. for 3 months, at $3\frac{1}{2}$ per cent.?

2. A clock has two hands, turning on the same centre; the swifter makes a revolution in 5 hours, the slower in 9. If they start from the same point, when will they next meet?

3. Prove Napier's rules for solving a right-angled spherical triangle, in the two cases, in which one of the sides is made the middle part.

4. Prove, that the tan. + cotang. of an arc is equal to twice the cosec. of double the arc.

5. The roots of $x^3 - 7x^2 + 14x - 8 = 0$ are in geometrical progression. Find them.

6. Find a number greater than the greatest positive root, and also one less than the least negative root of the equation $x^3 - 4x^2 - x + 20 = 0$.

7. A lighter fluid, whose specific gravity is 3, rests upon a heavier, whose specific gravity is 7. A paraboloid, with its vertex downwards, rests between them, and sinks with $\frac{1}{4}$ of its axis in the heavier.—Find its specific gravity.

8. Two bodies, *P* and *Q*, each 1 lb. in weight, balance on a single pulley. An ounce weight is added to *P*. How long is it in descending through 12 feet? and what velocity does it acquire?

9. A set of 5 balls, imperfectly elastic, are in a

geometrical progression, whose common ratio is 2. The force of elasticity is to the force of compression :: 3 : 2. Compare the velocity of the first with that communicated to the last.

10. The tube of a barometer is 33 inches long. A quarter of an inch of air was left in at the time of inversion. When the mercury in this barometer stands at 28 inches, what is the true standard altitude?

11. Find the latitude of the place, at which the sun sets at 10 o'clock on the longest day.

Tuesday Afternoon.—Mr. D'Oyly.

1. Find two numbers, such, that their difference may be 4, and that twice their product may equal the cube of the less.

2. A curve has its subnormal : (abscissa)^3 :: $2a^2$: 1. Find the equation to the curve; draw a tangent to it, and find its area.

3. Find the focal length of a double convex lens; and shew that in a glass lens, with surfaces equally curved, it is equal to the radius of either surface.

4. When the moon's elongation is 60°, delineate her phase, and prove the area of the enlightened part to be $\frac{1}{3}$ the area of the dark part.

5. A comet is in the perihelion of a given ellipse. Compare its velocity with the velocity it would have in a parabola, at the same perihelion distance.

6. An object is placed before a double concave lens, at the distance of 5 feet, and has the linear magnitude of its image 7 times less than its own.—Find the focal length of the lens.

7. Sum the series

$$\frac{1}{3} - \frac{1}{3 \cdot 2} + \frac{1}{3 \cdot 2^2} - \text{\&c. ad infin.}$$

also sum n terms of $1^2 + 2^2 + 3^2 + 4^2 + \text{\&c.}$

8. Find the fluents of

$$x^3 \dot{x} \sqrt{(1-x^2)}, \text{ of } \frac{x\dot{x}}{\sqrt{(1-x^4)}}, \text{ and of } \frac{z\dot{z}}{1+2az+z^2},$$

where a is less than unity.

9. Find the relation of x to y in $\sqrt{(1+y^2)} \times (x\dot{y} + y\dot{x}) = y\dot{y}$.

10. If the 4th power of the periodic times in different circles varies as the cube of the velocities, find how the force, periodic time, and velocity, vary in terms of radius.

11. When a body describes the reciprocal spiral, prove that it makes equal approaches to the center in equal times; and find the area contained between any two distances.

12. In what part of the moon's orbit does the ablatitious force bear to the addititious the proportion of 4 to 3?

Mr. D'Oyly.

FIFTH AND SIXTH CLASSES.

1. Find the greatest common measure of 720, 336, and 1736.

2. Prove the rule for finding the square root of a binomial surd, and extract the square root of the surd $5 - 2\sqrt{6}$.

3. Shew that in a plane triangle, the base : sum of sides :: difference of sides : difference of segments of base.

4. Find the value of x in $3x + 2\sqrt{x} - 1 = 0$.

5. An inclined plane has its length 6 times its height: How long will a body be in falling through 20 feet? and what velocity will it acquire?

6. A body weighing 6 lbs. in air, weighs 2 lbs. in water; another weighing 7 lbs. in air, weighs 4 lbs. in water. Compare their specific gravities.

7. Prove that the periodic times in all ellipses round the same center are equal.

8. Sum $\frac{2}{10} - \frac{2}{100} + \frac{2}{1000} -$ &c. ad infin.
and $5 + 7 + 9 + 11 +$ &c. to 50 terms.

9. Find the fluxion of

$$\frac{x}{\sqrt{(1+x^2)}}, \text{ and the fluent of } \frac{a\dot{x}}{\sqrt{(2ax - x^2)}}.$$

10. A short-sighted person, who cannot see distinctly beyond 3 feet, wishes to see an object at 14 feet distance. What sort of glass must he use, and what must be its focal length?

Evening Problems.—Mr. D'OYLY.

1. Find the exact value of the decimal .012323 &c. of a pound.

2. Let $x^2y^2+6xy=16$, and $\sqrt{\frac{1}{4x}}=\frac{1}{2}\sqrt{\frac{1}{y-1}}$, Find corresponding values of x and y.

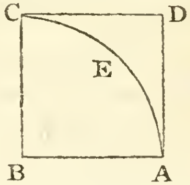
3. The roots of $8x^3-6x^2-3x+1=0$ are in harmonical proportion. Find them.

4. Prove that the perpendicular ascent of an heavenly body is always quickest on the prime vertical.

5. A cylindrical vessel of given diameter is filled with fluid to a certain height; its height is divided at some point. What must be the height of the fluid, and where the point of division, so that the pressures on the base, on the upper surface, and on the lower, may all be equal?

6. Find the attraction of a particle placed in the vertex of a paraboloid, the attraction on each particle being reciprocally as (dist.)^2

7. The equation to a curve is $y\sqrt{(2ax-x^2)}=ax$.

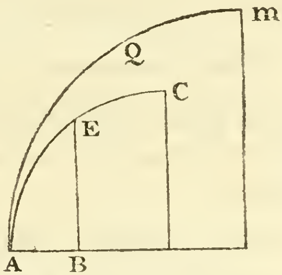


Find its area; and prove, that when x is equal to a, the whole area equals the excess of $ABCE$ above

ADCE; *AEC* being a quadrant, and *ABCD* a square on *AB*.

8. A body weighing 2lbs. is projected with a velocity 20, at an angle of 60°; another, weighing 3lbs. is at the same time projected at an angle of 30°, with a velocity 25. Trace the motion of their common center of gravity; find the height to which it mounts, and the distance at which it strikes the horizontal plane.

9. *AEC* is a quadrant; the curve *AQm* is traced



by taking its abscissa always = arc *AE*, and its ordinate = twice sine *EB*. Draw a tangent to the curve at any point, and find its area for the whole quadrant.

10. The chances that each of two individuals will die in a certain time are respectively $\frac{5}{6}$ and $\frac{4}{5}$. Find the two probabilities, that they will not both be alive, and that they will not be both dead, at the end of that time.

11. Explain the method of transforming an equation, in which x flows uniformly into one, in which

y flows uniformly; and by this method solve the equation $\dot{x}\dot{y} - x\ddot{y} = x\dot{y}^2$.

12. A weight, suspended by a string of given length, is whirled round in a conical motion; that is, it describes, parallel to the horizon, the base of a cone, of which the vertex is the point of suspension. The tension of the string being 3 times the natural weight of the body, in what time does it revolve?

13. A body is projected from an apse with a velocity, which bears to the velocity, in a circle at the same distance, the proportion of 1 to $\sqrt{3}$, force varying as $\frac{1}{D^7}$. Find the angle which it describes round the center of force, before it falls into it; and compare the time with the periodic time in a circle at the same distance.

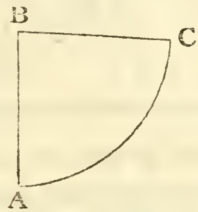
14. A straight line is placed at the distance of 9 inches before a double convex lens, whose focal length is 6 inches. Prove that the image will be a conic section, and determine its major and minor axis.

15. A cylindrical bent tube is partly filled with a fluid, whose surfaces rise and fall alternately. Shew that this fluid librates twice, in the time that a pendulum, whose length equals twice the line of fluid, oscillates once.

16. A chain of uniform thickness, hanging freely, and pliant in every part, forms itself into a cycloid. Required the law of its density and weight.

17. The quadrant AC is a thin rod, which re-

volves on the axis AB perpendicular to the horizon. A ring moves freely on the rod. With what velo-



city must the point C revolve, in order that the ring may always remain at the middle of the arc?

18. A pendulum, vibrating in a certain time at the pole of a sphere, vibrates once less in 100 times, when carried to a place 30° from its equator. In what time does the sphere revolve round its axis?

19. Given a star's aberration in latitude and longitude; find the aberration, in declination and right ascension; and explain where a star must be situated, so as to have the same aberration in latitude and declination.

20. Two globes, of equal weights and diameters, are placed in contact, on a straight rod which passes through both their centers; the point of suspension is distant one diameter from the vertex of the upper one. Find their common center of oscillation.

21. Find fluents of $v^3x^2\dot{x}$, where v is hyp. log of x; of $\frac{\dot{x}}{x\sqrt{(1+\sqrt{x})}}$, and of $vx\dot{x}$, where $v=$ hyp. log. $\frac{a+x}{a-x}$.

22. Sum the following series:

$$\frac{1}{2} - \frac{1}{2.2^3} + \frac{1}{2.2^6} - \text{\&c. ad infin.}$$

$$\frac{2}{1.3.4} + \frac{3}{2.4.5} + \frac{4}{3.5.6} \text{ \&c. to } n \text{ terms;}$$

and, $1^2 + 4^2 + 7^2 + 10^2 \ldots\ldots$ to n terms.

23. The powers of the sun and moon to raise a tide being respectively given, and the moon's elongation from the sun. Find the place of the compound high tide.

24. A man descends from an height in the atmosphere, suspended from the center of a plane circle, whose diameter is d. The weight (in air) of himself and the machine is w lbs.; a cubic foot of air weighs every where $1\frac{1}{5}$ oz. and gravity is constant; find the greatest velocity he can acquire, and shew that he can attain this velocity in no finite time.

1808.

First Morning —Mr. D'Oyly.

Monday, January 18, 1808.

FIRST AND SECOND CLASSES.

1. Prove that the difference of the tangent and co-tangent of an angle is equal to twice the co-tangent of double the angle.

2. Prove Napier's rules for the solution of a right-angled spherical triangle, in the two cases, in which the complement of the hypothenuse is made the middle part.

3. When a pendulous body oscillates in a cycloid, beginning at the highest point, compare the tension of the string at the lowest point, with the tension caused by the natural weight of the body.

4. Find the actual velocity and periodic time of a body revolving at its distance of two of the earth's radii above its surface.

5. A straight rod is suspended at a point, distant $\frac{1}{4}$th of its length from its extremity. Compare the time of its oscillation in this case, with the time of its oscillation when it is suspended at its extremity.

6. Given $a\dot{z} = (by + a^2)^{\frac{1}{2}}\dot{y}$, where z is the arc of a curve, and y the ordinate; find the relation between the ordinate and abscissa.

7. Find the centre of gravity of a given sector of a circle.

8. If two bodies are connected by a lever void of gravity, and a force acts perpendicular to the line which joins them, at a point which is not their center of gravity; find the center of initial rotation.

9. A cylinder, whose radius = 2 feet, and whose weight = 100 lbs. is moveable on its axis. A weight of 1 lb. is suspended at the distance of half its radius from the axis. How long must this weight act, to generate a motion of 10 revolutions in 1″?

10. Affect $\underset{-3}{v}\ \underset{-1}{v}\ \underset{2}{v}$ with v, and reduce it to succeeding values of v.

11. Find fluents of $(\sin. z)^{\frac{1}{2}}\dot{z}$, of $\dfrac{x^m \dot{x}}{(\log. x)}$, and of $\dfrac{\dot{x}}{1-x^4}$.

12. Sum $\dfrac{5}{1.2} \times \dfrac{1}{3} + \dfrac{7}{2.3} \times \dfrac{1}{9} + \dfrac{9}{3.4} \times \dfrac{1}{27} +$ &c. to n terms;

$$\text{and } \frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \text{ \&c. ad inf.}$$

13. Compare the chance of throwing, with a single die, an ace at least 3 times in 5 trials, with the chance of throwing it 3 times exactly in the same number of trials.

First Afternoon.—Mr. Woodhouse.

1. Compare the interest on £350. 15*s.* at 4½ per cent. with the interest on £450. 15*s.* at 3½ per cent.

2. Prove, 1st, That three perpendiculars from the angles of a triangle upon the opposite sides intersect each other in the same point.

2dly, That three lines drawn from the three angles of a triangle, and bisecting the opposite sides, intersect each other in a point which is the center of gravity of the triangle.

3dly, That three lines drawn from the three angles of a triangle, and bisecting those angles, intersect each other in a point, which is the center of an inscribed circle.

4thly, That three lines bisecting the three sides of a triangle, and drawn perpendicularly to the sides, intersect each other in a point which is the center of a circumscribed circle.

3. In what point of the cycloid will the force stretching the string, and the force accelerating the body, be equal?

4. If the perihelion distance of a comet moving in a parabolic orbit be 64, the earth's mean distance being 100, compare its velocity at the extremity of the latus rectum with the earth's mean velocity.

5. Compute the value of the tangent of 15°.

6. By what method can the planets' distances from the sun be computed?

7. Find the fluxion of $(1 - x^{\frac{1}{2}} + x^{\frac{1}{3}})^{\frac{4}{5}}$,

and the fluent of $\dfrac{\dot{x} + \frac{1}{3} x \dot{x}}{\sqrt{(1 + x^2)}}$.

8. Place an object 100 feet before a concave spherical reflector, of which the radius is 10 feet; then find the distance between the principal focus and the focus of reflected rays.

9. Define the longitude of a place, and explain by what methods it can be determined.

10. Find the present value of a perpetuity of £5. per annum, to be received at the end of seven years, the rate of interest being 5 per cent.

11. Investigate an expression, from which a pendulum, losing or gaining n seconds in 24 hours, may be shortened or lengthened.

12. Explain what is meant by the Aberration of Light?

If from such aberration we perceive the apparent places only of stars, by what means can their real places be assigned?

13. What is the difference between the latera recta of a parabola and ellipse having the same least distance = 1, the axis major of the ellipse being 300?

14. Given the logarithms of 20 = 1.30103000
of 30 = 1.47712125
Deduce thence the logarithm of 360.

15. Explain the construction of the horizontal sun-dial; and compute, for the latitude of Cambridge, the hour-angle between 2 and 3.

16. If bodies are imperfectly elastic, shew that the sum of the product of each body into the square of its velocity, before impact, is greater than the sum of the product of each body into the square of its velocity after impact.

17. If the sun's apparent diameter at the extremity of the latus rectum were to his apparent diameter at perigee as 100 : 101, what would be the excentricity of the solar ellipse?

18. Define the centrifugal force; find an expression for the law of its variation; and compare it with the centripetal force which acts upon a body moving in the logarithmic spiral.

19. Find the nature of the curve, in which the square of the ordinate is a mean proportional, between the area and a given quantity b.

20. Let a body be projected at an angle of 45^0 with a velocity acquired down the axis of a cycloid. Compare the time of flight with the time of an oscillation.

21. Sum the arithmetical series

$$1+5+9+13 \text{ \&c. to } n \text{ terms.}$$

22. State Napier's rules; and prove their truth, in the case in which the complement of the hypothenuse is the middle part.

23. Find the attraction of a corpuscle situated in the axis of a cylinder towards the cylinder, the particles of which attract according to the law of the inverse square of the distance.

Evening Problems.—Mr. WOODHOUSE.

1. Define discount; investigate a formula for computing it; and apply such formula to the computation of the discount on £100. due at the end of 3 months.

2. Find the sum of n terms of a series of polygonal numbers, which numbers are formed, by assuming any arithmetical series that has its first term 1, and difference a whole number, and by making, generally, the m^{th} polygonal number equal to the sum of n terms of the arithmetical series.

3. By Nautical Almanack, the moon's right ascension was at Midnight, March 10, = 4°. 56′,
Noon, March 11, = 10°. 34′,
Midnight, March 11, = 16°. 18′.
It is required to find the right ascension March 11, 3^h.

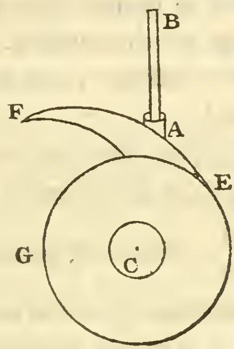
4. The axis of the solar ellipse having a yearly progressive motion of 1′. 2″, and the longitude of the perigee being in 1750, 278°. 37′. 15″; it is required to find the epoch, at which the axis was perpendicular to the line of the equinoxes.

5. From the two ends of a vertical line, two bodies are, at the same instant, projected towards each other, with velocities v, v'. Required their distance, when half the time, in which they would meet, is elapsed.

6. If a power P descending vertically by means of a string passing over a fixed pulley, elevates a

weight W; what will be the pressure sustained by the axis of the pulley?

7. Let the wheel EG revolve round an axis at C. Required the nature of its tooth or *wing* EF, that



shall elevate vertically the piston AB through equal spaces, whilst the wheel revolves through equal angles.

8. Given the velocity of sound; find the horizontal range when a ball at a given angle is so projected towards a person, that the ball, and sound of the discharge, reach him at the same instant.

9. The late comet passed its perihelion Sept. 21, 1807; and the perihelion distance was .64, the earth's mean distance being 1. Required the computation of the velocity with which it is now moving, January 18, 1808; the orbit being supposed to be parabolic.

10. With three dice in one throw, compare the chance of throwing the number 10 with the chance of throwing the number 5.

11. Find the fluent of the fluxional equation

$$\frac{\ddot{y}}{\dot{t}^2} + a^2 y = 0.$$

and of $\dot{\theta}$ sin. $m\theta$. sin. $n\,\theta$. (radius = 1.)

12. Let a square be immersed in a fluid, with its plane perpendicular to the surface. Compare the pressures on the two halves of the square, made by drawing its diagonal.

13. If the fluent of the fluxional equation $p\dot{x} + q\dot{y} = 0$ can be assigned, p and q being functions of x and y then $\frac{\dot{p}}{\dot{y}} = \frac{\dot{q}}{\dot{x}}$. Required proof.

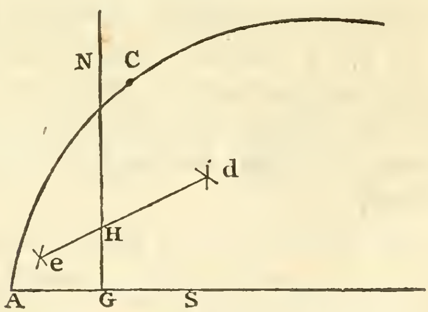
14. Compare the velocity of the moon in octants, with her velocity in syzygies, according to the principles of the 11th section.

15. Find, by the method of increments, the sum of

$$\frac{1}{1.2.3.4} + \frac{3}{2.3.4.5} + \frac{6}{3.4.5.6} + \frac{10}{4.5.6.7} + \text{\&c.}$$

the numerators being the second order of figurative numbers.

16. If C be the place of a comet in its parabolic



orbit, A the perihelion, S the sun, G the middle point between A and S, GN a perpendicular to AS; then, if from the centers C and S, and with any radius, circular arcs be described intersecting each other in e and d, and ed be joined, a point H in GN is determined, such, that GH shall be proportional to the time of the comet's moving through the arc AC. Required proof.

17. Let d be the distance of the focus of incidence from the center of a concave reflector, and let a ray be incident on the reflector 60^0 from its vertex. It is required to find the distance, in the axis of the reflector, between the geometrical focus and the intersection of the reflected ray with the axis.

18. If a pendulum, vibrating through an arc of 2 degrees on each side of the vertical, keep true time, find nearly the error of time introduced by making it vibrate through $2^\circ.\ 10'$.

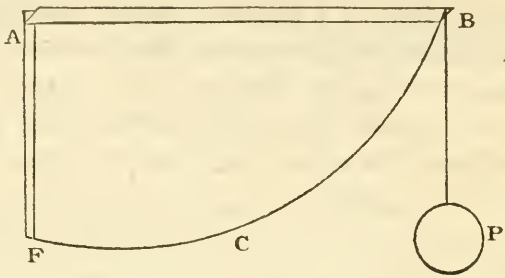
19. Express the cos. $4x$ in terms involving powers of cos. x.

20. Investigate the nature of the curve, of which the normals bear the same relation to the parts of the abscissa intercepted between the origin of the abscissas and normals, that the ordinates of a common parabola bear to their corresponding abscissas.

21. Suppose a piston, closely fitting a vertical tube, to descend by its weight, the tube being filled with an elastic fluid, and closed at the end opposite to the piston; find the velocity of the piston after a descent through a space S, not considering the effect of friction.

22. If a body be made to descend obliquely along the concave surface of a vessel, the axis of which is perpendicular to the horizon; then the fluxion of the area, projected upon a plane perpendicular to the axis, is a constant quantity. Required demontion.

23. *BCFA* is a piece of timber inserted into a wall at *AF*. If a weight *P* be appended to the end



B, what ought to be the form of the under-bounding surface *BCF*, that the strength of the beam may be every where the same; the weight of the beam being supposed to have no effect in producing fracture?

24. $x^3 - 2x^2 + 1 = 0$. Deduce the equation of which the roots are the cubes of the roots of the original equation.

Tuesday Morning Problems.—Mr. Woodhouse.

FIRST AND SECOND CLASSES.

1. The rate of interest being 5 per cent. in what number of years, at compound interest, will £1. amount to £100.? (Log. 1050 = 3.0211893.)

2. Prove, that tan. 50 = tan. 40 + 2. tan. 10.

3. Find the algebraic relation between v and z that satisfies this equation

$$\frac{\dot{v}}{\sqrt{(Av^2+Bv+C)}}+\frac{\dot{z}}{\sqrt{(Az^2+Bz+C)}}=0.$$

4. Prove, that

$$\text{hyp. log. } x = x - x^{-1} - \frac{x^2-x^{-2}}{2} + \frac{x^3-x^{-3}}{3} - \&c.$$

5. If $e^y = \frac{e^x+1}{e^x-1}$, find the length of the curve; y and x being its ordinate and abscissa, and e the number whose hyp. log. is 1.

6. Required the time of the passage of a comet from its perihelion, through 45° of anomaly; the perihelion distance being 1.

7. In the 10th section, if the diameter of the wheel be equal to the radius of the globe, the cycloid within the globe, or the *hypocycloid*, becomes a right line passing through the center. Required proof.

8. Explain the principle and use of the common Vernier or Nonius.

9. Apply the method of increments to sum n terms of the series

$$\frac{1}{1.2.3}+\frac{2^2}{2.3.4}+\frac{3^2}{3.4.5}+\frac{4^2}{4.5.6}+\&c.$$

$$1 + 2.5 + 3.5^2 + 4.5^3 + \&c.$$

10. If the sun's declination on the meridian of Greenwich

were, October 5, $4^{\circ}.\ 28'.\ 36''$,
and October 6, $4^{\circ}.\ 51'.\ 46''$,

what would be the declination at 2 o'clock, October 5, at any other place 45^{0} west of Greenwich?

11. Prove that the times of moving from the perihelion to the extremity of the latus rectum, in different parabolas, vary in the sesquiplicate ratio of the perihelion distances.

12. If, with a force varying $\frac{1}{(\text{dist.})^2}$, a velocity which is to velocity in circle as $\sqrt{3} : \sqrt{2}$, at an angle θ, and at a distance d, a body be projected; find the excentricity of the orbit described.

13. Let the luminous point be the extremity of the diameter of the circular reflector; find the length of the caustic, its cusp, and the density of rays in its cusp.

Mr. Woodhouse.

FIFTH AND SIXTH CLASSES.

1. If $\frac{3}{4}$ of a yard cost 9*s*. 4*d*. what number of yards will £100. purchase?

2. Solve the equation $1 - \frac{2-x}{x} = \frac{x+2}{2} - 1$,

and of the two expressions $x^3 + y^3$
$x^2 y + x y^2$.

Shew which is the greater.

3. With what velocity must a body be projected from a tower, in a direction parallel to the horizon, so that it shall strike the ground at a distance from the foot of the tower, equal half the height of the tower?

4. What is the present value of the compound interest of £100. to be received five years hence?

5. Compare the angular velocity of the earth at its mean distance, with its angular velocity in perihelion.

6. Find the fluxion of $\frac{z}{1+z}$, and the fluent of $x^{\frac{3}{5}}\dot{x}(1+ax^{\frac{6}{5}})^{-\frac{1}{3}}$.

7. Find the focal length of a spherical drop of water, the radius of which is $\frac{1}{10}$th of an inch.

8. If the periodic time of Mercury be to the periodic time of the earth as 4 : 11, required the time from conjunction to conjunction.

9. In what time will a weight move an equal weight along an inclined plane, the length of the inclined plane being 12 feet, and the height 10 feet?

10. Required the specific gravity of a body that floats in 2 fluids, of which the specific gravities are a and b, when the n^{th} part of the body is in the upper lighter fluid.

11. Given, in a rectilinear triangle, two sides and the included angle; find the third side.

Tuesday Afternoon.—Mr. D'Oyly.

THIRD, FOURTH, FIFTH, AND SIXTH CLASSES.

1. Find the exact value of the circulating decimal .212121 &c.

2. Find a number consisting of 2 places, such, that when it is divided by the difference of its digits, the quotient is 21; and that, when it is divided by the sum of its digits, and the quotient increased by 17, the digits are inverted.

3. The roots of the equation $x^3 - 14x^2 + 56x - 64 = 0$ are in geometrical progression. Find them.

4. Given the tangents of two arcs, find the tangents of their sum and difference.

5. Shew, that in a right-angled spherical triangle, if the legs are of the same affection, the hypothenuse is less than a quadrant; and, if the legs are of different affection, the hypothenuse is greater than a quadrant.

6. A body, projected at an angle of 15° to the horizon, ranges 100 feet on an horizontal plane.—How high would it rise, if it were projected straight upwards, with the velocity of projection?

7. The barrel of a condenser, and each barrel of an exhausting air-pump, is $\frac{1}{10}$th of the receiver, and the receiver is equal in both. Let each be worked 3 turns. How much is the density increased in the former, and how much is it diminished in the latter?

8. Prove, that parallax depresses an heavenly body in a vertical circle, and that the parallax at any altitude varies as the sine of the apparent zenith distance.

9. Let a point be taken in the axis of a given ellipse, half-way between its center and focus. Find the shortest line from this point to the circumference.

10. Prove, that if a body could fall from the surface of the earth to its center, it would acquire a velocity equal to that with which it would revolve in a circle at the surface; and find the actual velocity.

11. Find the fluents of

$$\frac{\dot{x}}{1+x^2}, \quad x^3 \dot{x} \sqrt{(1+x^2)}, \quad \frac{(a^2+x^2)^{\frac{1}{2}} \dot{x}}{a},$$

and $a^x x^2 \dot{x}$.

12. Prove that the mean addititious force in quadrature is to the moon's whole gravitation to the earth, as the square of the moon's periodic time to the square of the earth's periodic time.

13. Compare the side of an equilateral triangle inscribed in a circle with the radius of the circle.

14. Given the sine of 1′; explain how the sines of 2′, 3′, &c. may be found.

15. Solve the quadratic equation $5x = 3 - 2x^2$.

16. Find the 4 roots of the recurring equation $x^4 - \frac{5}{2} x^3 + 2x^2 - \frac{5}{2} x + 1 = 0$.

17. Deduce the equation to a parabola, and prove it to be a conic section.

18. A body is projected up an inclined plane, whose height is $\frac{1}{6}$th of its length, with a velocity of 30 feet per second. Find its place, and its velocity after 12 seconds.

19. A cylinder, whose height is 3 times its diameter, is filled with a fluid. Compare the pressure on the sides with the pressure on the base.

20. An object placed 4 inches before a double convex glass lens, has its image formed 9 inches from the lens on the same side. Required the focal length of the lens.

21. Find the attraction of a particle placed in the vertex of a cone, the attraction of each particle being inversely as $(\text{dist.})^2$

22. If the force varies inversely as $(\text{dist.})^2$, how far must a body fall externally and internally, to acquire the velocity in a circle?

23. Sum series

$$\frac{1}{2}-\frac{1}{2.3}+\frac{1}{2.3^2}-\frac{1}{2.3^3}+\text{\&c. ad infin.}$$

$$\frac{1}{2.4.6}+\frac{1}{4.6.8}+\frac{1}{6.8.10}\text{ to } n \text{ terms.}$$

24. Trace Cotes's first spiral; compare the velocity of a body in it with the velocity in a circle, and the time of its falling to the center with the periodic time in a circle.

Mr. D'Oyly.

FIFTH AND SIXTH CLASSES.

1. What fraction of half-a-crown is $\frac{1}{7}$ of $\frac{2}{11}$ of a guinea? and what is its exact value?

2. Extract the square root of $2 - 4\sqrt{(-2)}$.

3. What number is that, the treble of which, added to the double of its square root, is equal to 1.

4. Given the sines and cosines of two arcs; find the sine and cosine of their sum and difference.

5. Prove Cardan's rule for solving a cubic equation; and shew that it can only be applied where two roots are impossible.

6. A body is projected up an inclined plane, whose length is 10 times its height, with a velocity of 30 feet in 1″. In what time will its velocity be destroyed?

7. A plane surface moves in a fluid, in a direction perpendicular to its plane, with a certain velocity. Another plane, 4 times as large, moves in a fluid of 3 times the density, and with twice the velocity. Compare the resistances.

8. Given the latitude of the place, and the sun's declination; find the time of its rising.

9. Find the fluxion of

$\frac{(a^2 - y^2)^{\frac{1}{2}}}{y^{\frac{1}{2}}}$, and fluents of $\frac{\dot{y}}{1+y}$ and $\frac{2b\dot{x}}{x\sqrt{(x^2 - b^2)}}$.

10. Deduce the general expression for the centripetal force; and apply it to find the variation of the force in the logarithmic spiral.

11. Sum $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} +$ &c. to inf.

$1 + 2 + 3 + 4 +$ &c. to n terms.

12. Shew, that when the force $\propto$ dist. all times of descent to the center are equal; and that the velocity at any point is as the sine of the arch, whose versed sine is space described, the first distance being radius.

Evening Problems.—Mr. D'Oyly.

FIRST, SECOND, THIRD, AND FOURTH CLASSES.

1. Prove the rule for extracting the square root of a binomial surd; and apply it to extract the square root of $1-4\sqrt{(-3)}$.

2. The sun's declination being 16^0 north, required the latitude of the place where it rises in the north-east point, and also the time of its rising.

3. P and Q, each weighing 2 lbs. balance one another over a pulley; 1 oz. is added to P. How far will it have descended in 5″, and what velocity will it have acquired?

4. Compare the density of light in the sun's image, formed by a sphere of water, and by a plano-convex

glass lens of twice the aperture and twice the radius, the transmitting powers being supposed the same.

5. Find the fluents of $\frac{ax^2\dot{x}}{\sqrt{(1-x^2)}}$ of $\frac{\dot{v}}{(a^2+v^2)^2}$, and of $z^3\dot{y}$, where z is the circular arc, whose sine is y to radius a.

6. A vessel, whose shape is the frustum of a cone of given dimensions, is filled with a fluid, the smaller end being downwards, and is turned round on its axis with such a velocity, that all the fluid flies out. Required the velocity with which it turns round.

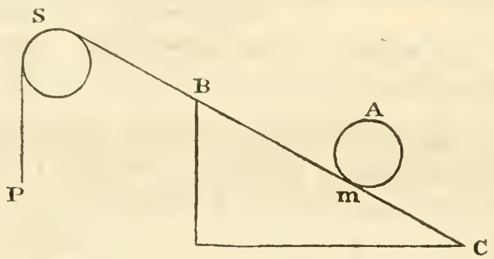
7. Approximate to a value of x in $x^3-x-50=0$; and shew on what the accuracy of an approximation depends.

8. While a body is revolving in a circle $\left(F \propto \frac{1}{D^3}\right)$ the absolute central force is suddenly doubled. Find in what time it will fall to the centre; and, when it has fallen half way, compare its velocity with that in a circle at the same distance.

9. The tangent of the angle, which the hour-lines for 3 and 4 o'clock make with one another on an horizontal dial, is to the tangent of the angle, which the hour-lines for 2 and 3 o'clock make with one another, in the given proportion of m to n. Find the latitude for which the dial is constructed.

10. A sphere A, whose weight is w, is made to

roll down an inclined plane BC, by means of a string wrapt round it, which is sustained by P, $P = \frac{w}{10}$. What must be the proportion of the height of the



plane to its length, that P may neither ascend nor descend?

11. When parallel rays fall on a spherical refracting surface of given radius, the ratio of the sines of incidence and refraction being known, find the longitudinal and latitudinal aberrations.

12. A curve is described, whose ordinate is the arc of a given circle, and whose abscissa is the corresponding sine. Find its area.

13. A thin rod of given length, weighing 1 lb. is suspended from its upper point, and has a weight $\frac{1}{2}$ lb. attached to it at $\frac{1}{3}$rd of its length from the point of suspension. Compare the time of its oscillation with this weight attached, with the time of its oscillation without it.

14. If the compressive force of the atmosphere varied as the logarithm of the density, and the density

varied inversely as the distance, what would be the law of gravity ?

15. If z be an arc of a circle to radius 1, shew that $\frac{1}{2}z = \sin. z - \frac{1}{2}\sin. 2z + \frac{1}{3}\sin. 3z - \&c.$

16. Integrate the fluxional equation

$$3x^2\dot{y} - 3ax\dot{y} = ay\dot{x}.$$

17. Prove that

$$a^x = 1 + \frac{A}{m} + \frac{2AX + A^2}{2m^2} + \frac{3AX^2 + 6A^2 + A^3}{6m^3} + \&c.$$

A and X being logarithms of a and x to modulus m.

18. A and B are playing a set at tennis. A wants two games, and B wants three; but A's skill is to B's in the proportion of 2 to 3. Find their respective chances of winning the set.

19. Find the integral of $\frac{1}{z}$.

20. Trace the curve, whose equation is $(x^2+y^2)^2 = x^2 - y^2$; find the value of x, when the curve is parallel to the axis; and determine the angle at which it cuts the axis, when the value of x is greatest. The Lemniscata, having a = 1.

21. A body moves in a logarithmic spiral in a resisting medium $\left(\text{density} \propto \frac{1}{\text{dist.}}\right)$ with a force varying as $\frac{1}{(\text{dist.})^{\frac{3}{2}}}$. It makes one revolution in t'', and approaches to the centre by 100th part of its first distance. Find the whole time of falling to the centre.

22. Sum series

$$1.4.5 + 2.5.6 + 3.6.7 + \text{\&c. to } n \text{ terms};$$

$$\frac{10}{1.2.4} + \frac{14}{2.3.5} + \frac{18}{3.4.6} + \text{\&c. to } n \text{ terms};$$

$$\frac{1}{1.4} - \frac{1}{3.6} + \frac{1}{5.8} - \text{\&c. to inf.}$$

23. The arch of a bridge being a cycloid, what must be the nature of the curve bounding the wall above, that all the parts of the arch may be in equilibrio?

1809.

First Morning.—Mr. D'Oyly.

Monday, January 16, 1809.

FIRST AND SECOND CLASSES.

1. Prove that if A and B are two angles, $\frac{\cos. A \times \cos. B}{r} = \frac{1}{2}\cos.(A+B) + \frac{1}{2}\cos.(A-B)$.

2. Find the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$, which are in arithmetical progression.

3. Given the horizontal parallax of an heavenly body, its right ascension, and declination; find its parallax in right ascension at a given time in a given latitude.

4. Compare the momentum of a circle revolving on one of its diameters with that of its circumscribing square.

5. Find the point of contrary flexure in a curve, whose equation is $x = (\text{hyp. log. } y)^3$.

6. A vessel in the shape of an hemisphere is filled with a fluid. Compare the time of its emptying itself through an orifice in its vertex, with the time of emptying through an equal orifice in its base.

7. Shew how the densities of planets may be compared.

8. Find the equation of the curve in which the sub-tangent $=b+y$; b being a constant quantity, and y the ordinate.

9. Find the integral of $\frac{1}{r^2}$.

10. Find the center of gyration of a spherical superficies revolving on one of its diameters.

11. Prove the sum of the series

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} +$ &c. continued ad infin. to be infinite.

12. Find fluents of

$$\frac{\dot{x}}{x^2.(x+a)}, \text{ of } \frac{x^2\dot{x}}{(x-2)(x-3)}, \text{ and of } \frac{x\dot{x}\log. x}{\sqrt{(a^2+x^2)}}.$$

13. There are four individuals, for each of whom the chance of dying in a certain time is $\frac{1}{10}$. Compare the chance, that two of them at least will be dead in that time, with the chance that exactly two will be dead.

Monday Afternoon.—Mr. Brown.

THIRD AND FOURTH CLASSES.

1. What is the interest of £260. for 18 months, at $4\frac{1}{2}$ per cent.?

2. What is the discount of the same sum, for the same time, at the same rate?

3. The arms of a bent lever are in the proportion of 2 : 1. What must be the angle at which they are inclined, that, when in equilibrio, the shorter arm may be parallel to the horizon?

4. Given R and r the radii of the surfaces of a convex lens, and $m : n$ the ratio of the sines of incidence and refraction; find the focal length, and shew it to be the same, which-ever way the rays come.

5. A pendulum gains n seconds a day. How far must it be elevated above the earth's surface to keep true time?

6. Solve the following equation: $x^3 - x^2 - 8x + 12 = 0$, which has 2 equal roots.

7. Find the respective bearings of London and Constantinople from each other.

8. Compare the velocity of a body at the vertex of a parabola, with the velocity in a circle at the distance of half the latus rectum.

9. Find how far a body must fall externally, to acquire the velocity in an ellipse, when the force tends to the focus.

10. Find the sum of the following series:

$\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} +$ &c. ad inf. and n terms of $1^2 + 2^2 + 3^2$ &c. by the method of increments.

11. Find the fluxion of ay^r; and the fluents of $\frac{\dot{x}}{\sqrt{(a^2+x^2)}}$, and of $\frac{x\dot{x}}{\sqrt{(2ax-x^2)}}$.

12. Find the area of the common parabola intercepted between the ordinates b and c.

13. Shew, generally, in the conic sections, that $\frac{QT^2}{QR}=L$.

14. A body is projected with a velocity less than that acquired from an infinite distance, the force varying inversely as the cube of the distance. Investigate the orbit, and find in what time the body will fall into the centre.

Mr. D'Oyly.

FIFTH AND SIXTH CLASSES.

1. If $2\frac{3}{4}$ yards of cloth cost $3s.\ 8\frac{1}{4}d.$ what will be the price of $7\frac{2}{5}$ yards?

2. Extract the square root of $1+4\sqrt{(-3)}$; and prove the rule by which it is done.

3. Find the value of x in $3x-\frac{x-4}{4}-4=\frac{5x+14}{3}$.

4. Find two numbers such, that their product may be 24, and the sum of their squares 52.

5. Given the sine of an arc $=s$ to radius 1; express its cosine, versed sine, secant, tangent, and co-tangent.

6. Shew that impossible roots enter an equation by pairs.

7. A body falls 20 feet down an inclined plane, whose height is one-sixth of its length. What is the time of its motion, and what its last acquired velocity?

8. A body is projected at an angle of 45° with a velocity of 50 feet per second. Find its horizontal range.

9. A body weighs 14 lbs. in vacuo, and 9 lbs. in water. Another weighs 8lbs. in vacuo, and 7lbs. in water. Compare their specific gravities.

10. A person who can see distinctly at the distance of 3 feet, wishes to see an object at 12 feet distance. What sort of glass must he use, and what must be its focal length?

11. Divide 30 into two such parts, that the square of the one multiplied into the other, may be a maximum.

12. Find fluents of

$$(a^2+x^2)^3 \times x\dot{x}, \text{ of } \frac{x\dot{x}}{a^2+x^2}, \text{ and of } \frac{a\dot{x}}{\sqrt{(2ax-x^2)}}.$$

Find also the fluxion of $\frac{y}{\sqrt{(1+y^2)}}$.

13. Sum 50 terms of the arithmetical series

$$1+3+5+ \&c.$$

Also $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+$ &c. ad infin.

14. If a body revolves in a circle, how far must it

fall externally and internally, to require the velocity in the circle: the force varying inversely as (dist.)^2?

15. Prove that in any curve, the centrifugal force : centripetal force :: $Sy^2 \times Pv : 2Sp^3$.

16. If the force varies as the distance, shew that the velocity of a falling body is as the sine, and the time as the arc; the versed sine being the space described.

17. Find the diameter of curvature in an ellipse, and the chords of curvature passing through the focus and through the center.

18. Define the following terms in Astronomy: The right ascension and declination, latitude and longitude of an heavenly body; the true anomaly and mean anomaly of a planet; diurnal parallax and annual parallax; solstitial colure, and equinoxial colure.

Evening Problems.—Mr. Brown.

1. What is the present worth of an annuity of £20. a year, to continue for ever, and to commence after 2 years; the rate of interest being 5 per cent.?

2. How far will the Tychonic system account for the phenomena of the heavenly bodies? What phenomena will it not account for?

3. A pendulum in the latitude of 60° oscillates once in a second, supposing the earth not to revolve round its axis. What must be the angular motion of the earth, that it may oscillate once in 3 seconds?

4. Compare the mean ablatitious force in a whole revolution of the moon with the force of gravity on the earth's surface.

5. Prove, in the latitude of Cambridge, that the diurnal path of a shadow formed on the horizontal plane by the vertex of a gnomon, is convex towards the north during the passage of the sun from the vernal to the autumnal equinox, and convex towards the south during his passage from the autumnal to the vernal equinox.

6. Find the probability of throwing two aces in three trials with a single die.

7. Shew in what position of the eye and object the image formed by a double convex lens will be magnified or diminished; and when it will be erect, and when inverted.

8. Construct the fluent of $\frac{\dot{z}}{1+2az+z^2}$, a being less than 1.

9. Find the time in which a body will describe a given space, when acted upon by a force varying inversely as the distance.

✓ 10. Find in MERCATOR's projection the length of the meridian intercepted between 30^0 and 60^0 of latitude.

11. Find the fluents of

$$\frac{\dot{x}}{(x-a)^2 \times (x-b)}; \text{ and of } \frac{\{a+\sqrt{(a^2-x^2)}\}\dot{x}}{x};$$

of $X^2\dot{x}$ where $X=$ hyp. log. of x. Also of

$$\frac{\dot{x}}{x^4-2x^3-x^2+2x};$$ the roots of the equation

$x^4-2x^3-x^2+2x=0$ being $-1, 0, 1, 2$.

12. The radii of a wheel and axle are in the proportion of $a:b$; a weight P, acting by means of a line on the circumference of the wheel, elevates another weight Q, suspended from the line which goes round the axle. Required the pressure on the axis.

13. At a given place, and a given time, find the angle between the ecliptic and horizon, the culminating point, and the height of the nonagesimal degree. When is the angle between the ecliptic and horizon the least?

14. Find the expression for $(\cos. x)^n$ in terms of the cosines of multiples of x.

15. In a given parabola to inscribe another parabola, whose area shall be a maximum; the vertex of the inscribed parabola being in the middle of the base of the other.

16. A given piece of gold is balanced by its weight of brass in vacuo. What addition must be made to the brass, that they may be in equilibrio when immersed in water?

17. Apply Taylor's Theorem to find the sine of an arc in terms of the arc itself.

18. Sum the following series:

$$1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \text{\&c. ad inf.}$$

$$\frac{1}{1.2.5} - \frac{1}{2.3.7} + \frac{1}{3.4.9} - \text{\&c. ad inf.}$$

$1^4 + 2^4 + 3^4 +$ &c. to n terms, by increments.

19. Given the lengths of two degrees in different latitudes; find the ratio of the polar and equatorial diameters.

20. The sum of the m^{th} powers of the roots of the equation $x^n - 1 = 0$ is n, whenever m is equal to n or to any multiple of n; but in all other cases $= 0$. Required a proof.

21. An iron globe descends in water by the force of gravity; find the greatest velocity it can acquire, and the time of describing a given space from rest.

22. A stone is whirled round horizontally by a string 2 yards long. What is the time of one revolution, when the tension of the string is 4 times the weight of the stone.

23. Transform the equation $x^n - px^{n-1} + qx^{n-2}$ &c. $= 0$, whose roots are α, β, γ, &c., into one whose roots are $(\alpha-\beta)^2, (\alpha-\gamma)^2, (\beta-\gamma)^2$, &c.; and shew the use of such a transformation in finding impossible roots.

Tuesday Morning Problems.—Mr. Brown.

1. Find the value of the decimal .346565, &c. in a vulgar fraction.

2. Shew how the latitude and longitude of a star may be found.

3. Two equal weights, P and Q, are connected by a string passing over a fixed pulley; what weight added to P will cause it to acquire in 6 seconds a velocity of 48 feet? and through what space will it have fallen in that time?

4. If $A+B+C=m\pi$, (m = any whole number, $\pi=180^0$); then, tan. A + tan. B + tan. C = tan. $A\times$ tan. $B\times$ tan. C.

5. Find the whole fluent of $\frac{x^m \dot{x}}{\sqrt{(a^2-x^2)}}$, when $x=0$, and m is an even number. Find the fluents of $\frac{\dot{z}}{\text{sin. } z}$, and of $\frac{y^{\frac{1}{2}}\dot{y}}{\sqrt{(r^3-y^3)}}$.

6. Find the ratio between the sines of incidence and refraction out of a certain fluid into air; so that the eye, which can just see the extremity of the base of a given cylindrical vessel when empty, may see the center of the base when it is three parts full.

7. Determine the ratio of the diameter and altitude of a cylindrical vessel open at the top, which shall contain a given quantity under the least possible internal superficies.

8. Find the expression for the proportion between the centripetal and centrifugal forces in any curve; and apply it to the reciprocal spiral.

9. Find the sum of the first n terms of

$\frac{1}{1.2.4} + \frac{1}{2.3.5} + \frac{1}{3.4.6}$, by increments;

and of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$, &c. ad inf.

10. Find the increment of the moon's velocity by the tangential force, while she moves from quadrature to syzygy.

11. Shew that, if the resistance in any medium be proportional to the velocity, the oscillations in a cycloid will be isochronous.

12. If a be root of the equation, $x^n - px^{n-1} + qx^{n-2} + \&c. \ldots \pm w = 0$; shew that the equation is divisible by $x - a$, without considering it as composed of the factors $(x-a).(x-b).\&c.$

13. Suppose a cylindrical rod, whose length (l) and weight (w) are given, to be suspended at a distance (d) from one of its ends. If a weight (a) be attached to the lower end, what weight must be attached to the upper, that the time of oscillation may be the same as before?

Mr. D'Oyly.

THIRD AND FOURTH CLASSES.

1. Find two values of x in the equation $x + \frac{24}{x-1} = 3x - 4$.

2. Given the three sides of a plane triangle; explain how the three angles may be found.

3. Divide the number 100 into two such parts, that the square of the one multiplied into the square root of the other, may be a maximum.

4. Find between which of the roots of the equation $x^3 - 7x^2 + 7x + 10 = 0$, the number 3 lies.

5. A stone projected at an angle of 30° strikes the horizontal plane at the distance of 50 yards. With what velocity was it projected?

6. A pendulum vibrates 61 times in a minute. How must its length be altered, so that it may vibrate 60 times in a minute?

7. In the tube of a barometer 36 inches long, the mercury stands at $29\frac{1}{2}$ inches, when the standard altitude is 30 inches. How much air was left in at the time of inversion?

8. Find the principal focus of a given glass globe.

9. Find the area of the curve, in which the subtangent $= \frac{b}{1+y^2}$; y being the ordinate, and b a constant quantity.

10. Find fluents of

$$\frac{x^3\dot{x}}{x-a}, \text{ of } y^2\dot{y}\sqrt{(a^2-y^2)}, \text{ and of } a^x x\dot{x}.$$

11. Explain the method of finding the hour of the day by observing the sun's altitude; the latitude of the place, and the sun's declination, being known.

12. If a hole were bored from the surface of the earth to its centre, what would be the velocity acquired at the centre, by a body which fell from the height of 1 radius above its surface; the force above the surface being inversely as $(\text{dist.})^2$, and below the surface being directly as dist.?

13. Having given the major and minor axis of an ellipse, and the force in the focus; compare the periodic time in the ellipse with the periodic time in a circle, whose radius = greatest distance in the ellipse.

Mr. Brown.

FIFTH AND SIXTH CLASSES.

1. Find the value of the decimal .124343 &c. in a vulgar fraction.

2. If $\frac{3}{4}$ of a yard of cloth cost $\frac{1}{4}$£., what will $12\frac{2}{3}$ yards cost?

3. At what time are the hour and minute-hands of a watch together between 2 and 3 o'clock.

4. Find the length of the cycloid.

5. Given $\left\{\begin{array}{l} x^3+y^3=72 \\ x+y=\ \ 6 \end{array}\right\}$ to find x and y.

6. Given the latitude of the place, and the sun's meridian altitude; to find the sun's declination.

7. Shew that the velocity in a parabola is to the velocity in a circle at the same distance as $\sqrt{2} : 1$.

8. Sum the series:

$$\frac{1}{3} - \frac{1}{6} + \frac{1}{12} - \frac{1}{24} + \&c. \text{ ad inf.}$$

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \&c. \text{ ad inf.}$$

9. Two columns of water and mercury communicate at the bottom of a bended tube. If the altitude of the mercury be 10 inches, what is the altitude of the water, supposing the ratio of their specific gravities to be 14 : 1?

10. What is the length of the longest day in latitude 45°?

11. Given the tangents of two arcs, to find the tangents of their sum and difference.

12. When the force varies inversely as the square of the distance, compare the velocity acquired through any space with the velocity in a circle at the same distance.

14. Shew, that the velocity in any curve, is equal to that which would be acquired with the same force continued uniform, through $\frac{1}{4}$ chord of curvature.

13. Find the field of view, and magnifying power, of the astronomical telescope.

14. Find the fluxion of

$$\frac{(a^4+x^4)^{\frac{1}{2}}}{x}, \text{ and the fluent of } \frac{2a\dot{x}}{a^2+x^2}.$$

15. Inscribe the greatest parallelogram in a given semi-circle.

Evening Problems.—Mr. D'Oyly.

1. Find the hyperbolic logarithm of 3, by applying Cotes's series for the hyp. log. $\frac{z+x}{z-x}$ to 3 terms.

2. Find an arc, which has the rectangle under its chord and cosine a maximum.

3. Approximate to a value of x in $x^3+2x-30=0$.

4. An object placed 4 inches before a double convex lens, has its image erect with respect to itself, and of three times its linear magnitude.—Required the focal length of the lens.

5. A body is projected at an angle of 60^0 to the horizon, with a velocity of 100 feet per second. What will be its time of flight before it strikes a plane inclined to the horizon at an angle of 30^0? and what will be its greatest elevation above that plane?

6. Investigate Sir I. Newton's rule for detecting impossible roots in an equation, and apply it to the equation $x^6+3x^4-4x^2-12=0$.

7. If a quadrantal arc is divided into any 3 parts, the sum of the products of the tangents of these parts taken two together, is equal to the square of the radius. Required proof.

8. The right ascensions and declinations of two stars being known, the time is observed when they come to the same azimuth. Shew how the latitude of the place may from hence be found.

9. Prove that the portion of the earth's surface included between the two circles of latitude 30° on each side of the equator, is half its whole surface; the earth being supposed to be a perfect sphere.

10. Find fluents of

$$\frac{x^6 \dot{x}}{x^2+a^2}, \text{ of } \frac{\dot{x}}{x^5 \sqrt{(x^2+a^2)}}, \text{ and of } \frac{\dot{z}}{\sin. z \cos. z}.$$

11. A chain of uniform thickness, hanging freely, forms itself into a cubical parabola. Required the law of its density and weight.

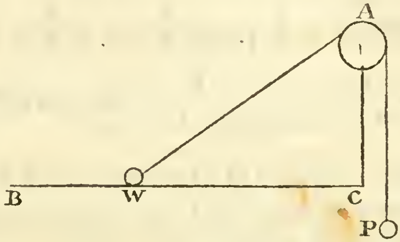
12. Let a plane circle oscillate on an axis, which passes perpendicular to its plane at the distance of half the radius from its center. At what other distance may the axis pass, so that the oscillation may be performed in the same time?

13. If a body revolve in an ellipse round the focus, required the point at which the angular velocity of the perpendicular on the tangent is the least possible.

14. If a body is revolving in a circle $\left(F \propto \frac{1}{D^3}\right)$, and half the central force is suddenly taken away; after what revolution will it fly off to an infinite distance?

15. The node of the moon's orbit being distant 30° from syzygy, the part of the ablatitious force, which acts perpendicular to the plane of the orbit, is found to be $\frac{1}{4}$th of its greatest value. What is the moon's place?

16. Let *P* and *W* be two bodies connected by a string passing over the pulley *A*; *P* hanging freely,



and *W* being sustained on an horizontal plane. What velocity will be acquired by *W*, while it is drawn by *P* through the given space *BC*.

17. Integrate the fluxional equation

$$a\dot{x} = \frac{xy\ddot{y} + x\dot{y}^2}{\dot{x}}.$$

18. A given parabola is placed with its plane and axis perpendicular to the horizon, and its vertex downwards. From what point of its highest ordinate must an elastic body be let fall, so that after impinging once on the curve, it may strike the vertex?

19. Find the increment of $z \times \log. z$.

20. Let a system of bodies be moveable round a vertical axis. At what distance from this axis must a given force act, so that the angular velocity communicated to the system in a given time, may be the greatest possible?

21. A person is throwing with two dice. What is the chance of his throwing size-ace, at least once in four trials?

22. Sum n terms of the series

$$1.2.5+2.3.6+3.4.7+\&c.$$

Also n terms of $\frac{2}{1.3.4}+\frac{3}{2.4.5}+\frac{4}{3.5.6}+\&c.$

And $\frac{1}{1.5}-\frac{1}{3.7}+\frac{1}{5.9}-\frac{1}{7.11}+\&c.$ ad infin.

23. Given the place of a luminary, and a situation on the earth's surface; find how much the superior tide exceeds the inferior in duration.

24. A beam of given length, having its perpendicular section every where a given parabola, projects horizontally from a wall. Compare its strength to support a weight at its end, when the vertex of the parabola is downwards, with its strength, when the vertex is upwards; the weight of the beam itself not being considered.

1810.

First Morning.—Mr. Walter.

Monday, January 15, 1810.

FIRST AND SECOND CLASSSES.

1. Prove the rule for the multiplication of decimals.

2. What number of degrees, minutes, &c. in the English scale corresponds to $32^0. 15'. 25''.$ in the French scale of 400^0 in one circumference, 100 minutes in a degree, and 100 seconds in one minute?

3. The quantities of air expelled, by successive turns, from the receiver of an air-pump, decrease in geometrical progression.

4. Given the distance of a planet from each of two stars, whose right ascension and declination are known. Find the right ascension and declination of the planet.

5. If any odd multiple of a quadrant be divided into any three parts, the sum of the co-tangents of the three parts will equal their product.

6. If from a given point a straight line be drawn touching a circle given in position; the straight line is given in position and magnitude.

7. Prove that the sub-contrary section of an oblique cone is a circle.

8. On the axis of a given cycloid, ordinates are drawn equal to the corresponding arcs of the cycloid; find the nature of the curve passing through the extremities of these ordinates, and its area.

9. The aspect of a wall is due south, and the sun is in the south-east with an altitude of 30^0. It is required to find the breadth of the shadow cast by the wall.

10. Find the sum of the series

$$\frac{a}{c}+\frac{a+b}{ce}+\frac{a+2b}{ce^2}+\frac{a+3b}{ce^3}+\text{\&c. ad inf.}$$

11. Given the capacity of the recess containing compressed air, the space which that air would occupy in its natural state, the diameter and weight of the ball, and the length of the barrel of an air-gun: find the velocity with which the ball leaves the gun, and the time of describing the barrel.

12. If a body be projected from an apse with the velocity acquired in falling from an infinite distance, find after how many revolutions it will fall to the center when the force varies in a higher ratio than the inverse cube of the distance.

13. Shew that the series $\sin. A+\sin. (A+B)+\sin. (A+2B)+$ &c. is a recurring series; and find the scale of relation.

14. A body being projected along any curve, and acted on by gravity, it is required to find an equation for determining the point where it will leave the curve.

Mr. Turton.

THIRD AND FOURTH CLASSES.

1. If $1\frac{1}{8}$th ell English cost $\frac{5}{6}$ths of a guinea, what will $2\frac{1}{3}$d ells Flemish cost; a Flemish ell being $\frac{3}{8}$ths of an English ell?

2. If an angle of a triangle $= 120^0$, shew that the square of the side subtending that angle is equal to the squares of the sides containing it, together with the rectangle contained by those sides.

3. Of two rays passing through a spherical refracting medium, shew that the deviation of the more remote from the center is greater than that of the less remote.

4. Find at what distance from a given sphere an eye must be placed so as to see an n^{th} part of its surface.

5. The plane of a cycloid, whose axis $= a$, is inclined to the horizon at an angle of 60^0. Find the time of descent down a chord drawn from the vertex to the extremity of the base.

6. Prove that in all the conic sections the chord of curvature through the focus $= L \times \dfrac{SP^2}{SY^2}$, and the

diameter of curvature $= L \times \frac{SP^3}{SY^3}$; where L is the principal latus rectum, SP the focal distance, and SY the perpendicular on the tangent.

7. If the force vary directly as the distance from the center, shew that the velocity of a falling body is to that in a circle at the same distance as the right sine to the cosine of a circular arc, whose radius is the greatest distance and versed sine the space fallen through.

8. Construct Galileo's Telescope; investigate its magnifying power, and find the linear magnitude of the greatest field of view.

9. Find the fluent of

$$\frac{dx^{3n-1}\dot{x}}{\sqrt{(a^n - x^n)}}, \text{ and of } \frac{dx^{3n-1}\dot{x}}{a^n - x^n}.$$

10. Find the roots of the equation $x^4 + 3x^3 - 7x^2 - 27x - 18 = 0$, two of which are of the form $+a$, $-a$.

11. If a body fall through a finite altitude AS, the force varying inversely as the square of the distance, and on AS a semi-circle ADS be described; prove that the area described by the variable radius SD is equal to the area uniformly described in the same time in a circle whose radius is the half of SA.

12. Through what space must a body fall internally to acquire the velocity in a circle, the force varying inversely as the distance.

13. Given the sun's meridian altitude, and his

midnight depression below the horizon: find the latitude of the place, and the sun's declination.

14. Given the major and minor axis of an ellipse, and the force in the focus. Compare the time in which a body would fall from the farther apside to the focus with the periodic time in the ellipse.

15. Prove that, in different circles, round different centres, when the forces vary inversely as the squares of the distances, the periodic times will vary in the sesquiplicate ratio of the radii directly, and the subduplicate ratio of the absolute forces inversely.

16. If the forces of attraction to each point of a spherical superficies vary inversely as the square of the distance, the attraction of the convex and concave part, on a *corpuscle* placed without it, will be equal.

Mr. Walter.

FIFTH AND SIXTH CLASSES.

1. Required the value of 2710 lbs. at 6*s*. 8*d*. per lb.

2. Required the interest of £547. 15*s*. at 3 per cent. simple interest, for three years.

✓ 3. A certain number, consisting of two figures, gives, when divided by the figure on the right, a quotient equal to 27, and remainder 2; but, if divided by 9, it gives a quotient equal to three times the figure on the right, and a remainder 2. Find the number. Curious

4. Find the value of x in $\sqrt{x} - 4 = \frac{259 - 10x}{4 + \sqrt{x}}$.

5. Prove, that if $A \propto B$, and $C \propto D$; then $AC \propto BD$.

6. Compare P and W, when they are in equilibrio on a plane whose inclination is 30^0, the power being supposed to act parallel to the base of the plane.

7. One body remains at rest in the circumference of a circle, whilst another describes that circumference. It is required to ascertain the curve described by their common centre of gravity.

8. Bisect a given semi-cycloidal arc.

9. Prove, that the horizontal range of a body, projected in the direction AI, is equal to $\frac{\text{sin. } 2 \angle IAC \times P}{2 \cdot \text{rad.}}$; and prove sin. $2 \angle IAC =$ sin. $2 \angle EAC$.

10. Solve the equation $x^3 - 5x^2 - 8x + 48 = 0$; two of whose roots are equal.

11. Sum the series

$$1 + \frac{1}{3} + \frac{1}{9} + \text{\&c. ad inf.};$$

and $1 - 2 + 4 - 8 +$ &c. to n terms.

12. Compare the pressure on the base of a cube, filled with water, with the pressure on the four sides.

13. $QC : qC ::$ sin. R : sin. I, when diverging rays are incident on a plane refracting surface.

14. Given the altitude of a known star, when it is on the prime vertical; find the latitude of the place.

15. Find the fluxion of

$$\frac{x}{\sqrt{(a^3+x^3)}}; \text{ also the fluent of } \frac{a\dot{y}}{\sqrt{(a^2-y^2)}}.$$

16. Prove, that the velocity of the earth continually increases from the aphelion to the perihelion distance.

17. Compare the velocity in an ellipse, at the greatest distance, with the velocity of a body revolving in a circle at the same distance.

Evening Problems.—Mr. TURTON.

1. Explain the nature of the stereographic projection of the sphere: and shew that angles on the plane of projection are equal to the original angles on the surface of the sphere.

2. Prove, by means of Newton's fourth Lemma, that the area of the common parabola is equal to two-thirds of its circumscribing rectangle.

3. Having given a rectangular piece of metal of uniform thickness, it is required, from one of its angles, to draw a line cutting off a triangle, in such a manner, that, the remaining trapezium being suspended by the obtuse angle, the parallel sides of the trapezium may remain horizontal.

4. It is observed, that two stars, whose right

ascensions and declinations are known, pass the prime vertical at the same instant. Required the latitude of the place of observation.

5. Given the latitude of the place, and the sun's declination: find at what time of the day the azimuth of the sun increases the slowest.

6. Investigate, as Cotes has done, the variation of the density of the atmosphere, supposing the force of gravity to vary inversely as the square of the distance from the earth's centre.

7. One root of the equation $x^3-4x^2-3x+12=0$, is of the form $\sqrt{a}$, where a is not a square number. Solve the equation.

8. A given paraboloid is perforated by a cylinder whose axis coincides with that of the solid. Required the dimensions of the cylinder, so that the part taken away may be equal to that which remains.

9. In a combination of wheels and axles, in which the circumference of each axle is applied to the circumference of the next wheel, and in which the ratios of the radii of the wheels and axles are those of 2 : 1, 4 : 1, 8 : 1, &c. there is an equilibrium when the power : the weight :: 1 : p. Required the number of wheels.

10. If the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \text{\&c.} = 0$, be in arithmetic progression, the least root will be $\frac{p}{n} - \frac{n-1}{n} \times \sqrt{\frac{\{(n-1).3p^2 - 6nq\}}{n^2-1}}$, and

the common difference $\frac{2}{n} \times \sqrt{\frac{\{(n-1).3p^2 - 6nq\}}{n^2-1}}$.

Required the investigation.

11. Given the major and minor axes of an ellipse. Required the radius of a circle described round the focus as a centre, in which the periodic time is equal to the time of moving through the aphelion, from mean distance to mean distance.

12. Construct Newton's telescope, and investigate its magnifying power.

13. Compare the quantity of water discharged by two equal parabolas in the side of a reservoir kept constantly full; one of the parabolas having its base and the other its vertex downwards, and the summits of both coinciding with the surface of the fluid.

14. Suppose that a body falls from a given altitude to a centre, of which the attractive force varies as the distance, and that the system moves in a direction perpendicular to the line of descent, with a velocity equal to the greatest velocity which the body could acquire in its fall; construct the curve traced out by the body, and then find its area.

15. If z be an integral increasing or decreasing unequally; then $\frac{1}{z_m} = \frac{1}{z} - \frac{\underset{\cdot}{z}}{z z_1} - \frac{\underset{\cdot}{z} + \underset{\cdot\cdot}{z}}{z_1 z_2} - \frac{\underset{\cdot}{z} + 2\underset{\cdot\cdot}{z} + \underset{\cdot\cdot\cdot}{z}}{z_2 z_3}$ – &c. continued to $m+1$ terms. Prove this.

16. Find the fluents of the following quantities, viz. $\frac{d\dot{x}}{x^3\sqrt{(a^2+x^2)}}$; $\frac{x^2\dot{x}}{(x-a)(x-b)(x-c)}$;

$(a^2+x^2)^m \times \dot{x}$, when the fluent of $(a^2+x^2)^{m+r} \times \dot{x} = A$; $z^n \dot{y}$, where z is a circular arc, and y its sine to radius 1.

17. Sum the following series:

$$\frac{1}{1.4.7}+\frac{1}{4.7.10}+\frac{1}{7.10.13} + \text{\&c. ad infin.}$$

$$2^2 + 5^2 + 8^2 + \text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1.2}+\frac{1}{5.6}+\frac{1}{9.10} + \text{\&c. ad infin.}$$

$$\frac{2}{1.3.3}+\frac{3}{3.5.3^2}+\frac{4}{5.7.3^3} + \text{\&c. to } n \text{ terms.}$$

18. Materials are to be raised, through a given altitude, by a wheel and axle whose radii are known; the power, which is given, being applied to the circumference of the wheel. Find the quantity raised at each ascent when the greatest quantity in the whole is raised in a given time; the inertia of the machine being neglected.

19. Find the area on the plane of the horizon that that is bounded by the shadow of a tower of given altitude, between the hours of 8 and 2, in a given latitude: the sun being in the equinoctial.

20. Find the relation of x to y in the equation $\dot{x}(a+bx+cy)=\dot{y}(d+ex+fy)$.

21. Let a sphere of given diameter be projected in a fluid whose specific gravity is to that of the body as 1 to n; having given the velocity of projection, it is required to find the velocity after describing any space and also the time of describing it.

22. Prove that the projection of the rhumb-line,

on the plane of the equinoctial, to an eye situated in the pole, is a logarithmic spiral; and hence determine the length of any arc of the meridian, on the planisphere.

23. Determine, as Newton has done, the path of a projectile in a medium in which the resistance varies as the velocity; the force of gravity being uniform and acting in parallel lines.

24. Determine the dimensions of a conic frustum, of given altitude, on which, when moving in a resisting medium, in the direction of its axis, with its less end foremost, the resistance will be equal to that on the base of a given cylinder, moving with the same velocity; and at the same time, less than the resistance on any other frustum of the same base and altitude.

Morning Problems.—Mr. Turton.

1. Define similar curves when referred to their axes; and prove that similar and conterminous arcs have a common tangent at the common point of termination.

2. If (a) and (b) be two sides of a trapezium that are parallel to each other; prove that the centre of gravity of the figure will divide a perpendicular to those sides into two parts that are to each other as $2a+b$ to $2b+a$.

3. If a body fall from a finite altitude towards a centre of force, and the time of falling vary as the

n^{th} power of the space fallen through; required the law of the variation of the force.

4. Resolve $\frac{1}{1-z^n}$ into trinomial fractions without the aid of fluxions; n being an even number.

5. If the refracting curve be the logarithmic spiral, and rays issue from the centre, investigate the nature of the caustic.

6. Find the force of elasticity, so that, in the case of direct impact, the sum of the products of each body into the *cube* of its velocity may be the same before and after impact.

7. If a weight P be suspended by an inflexible line, whose length is (a), to what point must a given weight p be attached, so that the pendulum may oscillate in the least time possible?

8. There is a small aperture, whose area is (m) at a given distance (a) from the bottom of a vertical cylinder filled with water. When full, the fluid falls on the horizontal plane, at the distance (b) from the base; and after (t) seconds at the distance (c) from the base. Required the content of the vessel.

9. Let $y = A + Bx^m + Cx^n + Dx^p +$ &c. where $A, B, C,$ &c. are constant quantities: then if $\dot{x}$ be given, $\dot{y} = \frac{\dot{y}}{\dot{x}} \times \dot{x} + \frac{\ddot{y}}{1.2.\dot{x}^2} \times \dot{x}^2 \,\dagger\, \frac{\dddot{y}}{1.2.3.\dot{x}^3} \times \dot{x}^3 +$ &c. † + omitted after $\dot{x}^2$

Required the investigation.

10. Find the fluent of

$$\frac{dz^{pn+\frac{1}{2}n-1}\dot{z}}{a+bz^n}; \text{ and also of } \frac{dz^{\frac{5}{2}n-1}\dot{z}}{\sqrt{(a+bz^n)}}.$$

11. Find the amount of £1. for the time (t), at compound interest at a given rate; interest being due every moment.

12. In a given latitude, at a given hour, and on a given day, the altitude and azimuth of a star are observed. Required its right ascension and declination.

13. Suppose the earth a perfect sphere, and that a pendulum whose length is (a) inches, vibrates seconds in latitude 60^0. What will be the length of a pendulum that vibrates seconds at the equator?

14. Deduce Cotes's construction of his first spiral, by means of Newton's general proposition in the 8th section.

Tuesday Afternoon.—Mr. Walter.

THIRD AND FOURTH CLASSES.

1. How much ready money can I receive for a note of £75. due 15 months hence, at 5 per cent. discount?

2. Find the values of x and y, when $xy = 63$, and $(x+y)^2 : (x-y)^2 :: 64 : 1$. Also the value of x in this equation $x^2 . (x+4) + 2x . (x+4) = 2 - (x+4)$.

3. Divide a given angle into two angles, such that their sines may be in a given ratio.

4. Express the side of a regular decagon, inscribed in a circle, in terms of the radius.

5. Two bodies, A and B, are projected perpendicularly upward with velocities (a) and (b). It is required to assign the highest point to which their common centre of gravity will ascend.

6. Find the roots of the equation $x^3 - 13x^2 + 50x - 56 = 0$, two of whose roots are in the ratio of $2 : 1$.

7. The diameter of a cylinder is 10 inches, and the diameter of an orifice in its base .025; also the height of the water in the cylinder is $8\frac{1}{24}$ feet. Required the time of emptying.

8. Given the apparent perpendicular depth of an object under the water, to find the direction in which a ball must be fired from a given point, so as to strike the object.

9. Sum the series

$$3 + \frac{1}{2} + \frac{1}{12} + \text{\&c. } ad\ infinitum.$$

Also, $1 . 3 . 5 + 3 . 5 . 7 + 5 . 7 . 9 +$ &c. to n terms.

10. Determine the equation of a curve by whose revolution a solid is generated equal, at all altitudes, to $\frac{3}{5}$ths of its circumscribing cylinder.

11. Find the centre of gravity of a bar whose density $\propto x^n$; x being the distance from the vertex.

12. A known star rises in the north-east point; find from this circumstance the latitude of the place.

13. Prove that V^2 in any curve : v^2 in a circle at the same distance $:: \frac{\dot{y}}{y} : \frac{\dot{p}}{p}$, where y is the variable distance, and p the perpendicular on the tangent.

14. How does the centrifugal force vary in different curves? and how does it vary in different parts of the same curve?

Mr. Turton.

FIFTH AND SIXTH CLASSES.

1. If $\frac{3}{5}$ yards cost £$2\frac{1}{4}$, find the value of $5\frac{7}{8}$ yards both by vulgar fractions and by decimals.

2. Prove that if a straight line stand at right angles to each of two straight lines in the point of their intersection, it will be at right angles to the plane that passes through them.

3. Define a rhombus; and prove that the diagonals of a rhombus bisect each other at right angles.

4. If (a) be the first term, and (b) the sum of three terms of a geometric progression, find the common ratio.

5. If the fluxion of $\frac{\sqrt{(ax)}-b}{\sqrt{(a-x)}}=0$, find the value of x.

6. Given three bodies A, B, C, and their distances from a plane; find the distance of their common centre of gravity from that plane, supposing A and B to be on one side of the plane, and C on the other side.

7. Given the velocity and direction of projection, find the greatest height of the projectile above the

inclined plane; and from the expression deduce the greatest height above the horizontal plane.

8. Shew that if a plane mirror recede from a fixed object, the image will recede twice as fast.

9. Explain the principle on which the Hydrometer is constructed, and demonstrate the proposition on which the construction depends.

10. Construct the common Astronomical Telescope, and investigate its magnifying power.

11. Given the latitude of the place, and the sun's declination; to find his azimuth at six o'clock.

12. Shew that the velocity in any conic section is to the velocity in a circle at the same distance in the subduplicate ratio of $\frac{1}{2}L \times SP$ to SY^2.

13. Find the fluents of the following quantities:

viz. $\frac{d\dot{z}}{(a+bz)^{\frac{m}{n}}}$; $\frac{x^5\dot{x}}{a^2-x^2}$; and $(a^2+x^2)^m \times x^2\dot{x}$.

14. Sum the series

$$\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\text{\&c. to } n \text{ terms,}$$

and *ad infinitum*.

15. Investigate the assumptions by which an equation $(x^n - px^{n-1} + qx^{n-2} - \text{\&c.} = 0)$ may be transformed into others wanting the second or third terms.

16. Given the earth's radius and the space fallen through in one second at its surface, find the periodic time in a circle at a given distance above the earth's

surface; gravity varying inversely as the square of the distance.

17. If a body whose elasticity is to perfect elasticity as m to 1 be let fall from a given altitude (a) above a perfectly hard horizontal plane, and rebound continually till its whole velocity is destroyed; find the whole space described.

18. A given paraboloid floats in a fluid with its vertex downwards; compare the specific gravities of the body and the fluid, supposing half the axis to be immersed.

Evening Problems.—Mr. Walter.

1. A cylindrical bar is suspended by a given point in a semi-circle, whose diameter is the bar. Find the inclination of the bar to the horizon, upon supposition that the semi-circle is devoid of weight.

2. Prove, from a property of the circle, that if four quantities are proportionals, the sum of the greatest and least is greater than the sum of the other two.

3. Given the area of any plane surface, it is required to find the content of a solid, formed by drawing lines from a given point without the plane, to every part of its surface.

4. The inclination of a perfectly smooth bank to the horizon is 30°, and a body is projected up the bank in a direction making an angle of 45°, with the intersection of the bank and horizontal plane. It is

required to determine the curve described by the body, and the spot where it will again meet the horizon.

5. If two curves have a common axis, and ordinates which are always in a given ratio to each other, then tangents drawn from the extremities of any corresponding ordinates will meet the axis in the same point.

6. The direction of a bridge is from east to west, and the sun in the meridian. The arches being supposed semi-circular, it is required to find the curve terminating that part of the surface of the water which is illuminated by the sun's rays passing through any arch.

7. It is required to express the cosine of an angle of a spherical triangle in terms of the sines and cosines of the sides.

8. If a body revolves in any curve whose equation is $ap=y^n$, y being the distance from the centre of force, and p the perpendicular on the tangent; it is required to find the equation of the curve of a star's apparent aberration, as seen from this body.

9. The roots of the equation $x^3-px^2+qx-r=0$, are a, b, c; transform it into one whose roots shall be $(a+b)$, $(b+c)$, $(a+c)$.

10. Required the position of the eye in a given line perpendicular to the horizon, so that the image of a given circle on the ground may also be a circle, when projected on a plane perpendicular to the horizon by lines drawn to the eye.

11. Find by the help of the common tables the logarithm of a number consisting of seven figures.

12. The roots of the equation $x^3 - px^2 + qx - r = 0$, are in harmonical progression. Find them.

13. Given the sun's declination, and the latitude of the place; find the path described by the shadow of a staff on an horizontal plane.

14. Sum the series $1.2.4 + 3.4.6 + 5.6.8 +$ &c. to n terms.

Also $\frac{1}{3} - \frac{2}{4} + \frac{3}{5} - \frac{4}{6}$ &c. to infinity.

And $\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} +$ &c. to n terms, and to infinity.

15. A has (p) counters, and B has (q); also the chance of A's winning a single counter from B is to the chance of B's getting one from $A :: a : b$. What is the probability that A will win all B's counters?

16. Explain the method of finding the velocity with which a bullet struck a pendulous body.

17. If a ponderous cylinder is put in motion about its axis by a weight p, descending through a space s; and, p being taken off, the moment thus generated be employed to elevate another weight q: the space through which the cylinder's motion elevates q can never be so great as $s \times \frac{p}{q}$, even if the effects of friction are neglected.

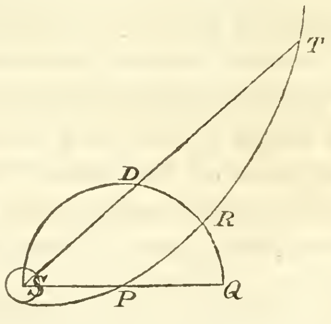
18. Find the fluent of $v^m x^n \dot{x}$, when v is the hyp. log. of $\frac{1}{x^2}$. Also the fluent of $\frac{x^2 \dot{x}}{x^2 - 3x + 2}$.

19. Find the relation of x and y, when $\frac{p\dot{x}}{x} + \frac{r\dot{y}}{y} = \frac{x^m \dot{x}}{ay^n}$, and also when $ay + \frac{b\dot{y}}{\dot{x}} + \frac{c\ddot{y}}{\dot{x}^2} = 0$.

20. Having given the velocity of projection, greater than that acquired by falling from an infinite distance, when the force $\propto \frac{1}{D^3}$, and also the distance from the centre of force; find the proper direction of projection so that any one of Cotes's three last spirals may be described.

21. Having given the fluents of $y\dot{x}, yx\dot{x}, yx^2\dot{x}, \ldots\ldots yx^{n-1}\dot{x}$; find the fluents of $\dot{A} = y\dot{x}$, $\dot{B} = A\dot{x}$, $\dot{C} = B\dot{x} \ldots\ldots \dot{R} = Q\dot{x}$, the number of the last equations being n.

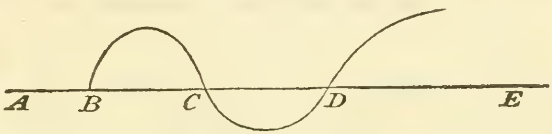
22. S is the pole of the logarithmic spiral, and P any point in the curve; if with centre P, and radius



PS, a circle be described, and QD taken equal to n times QR, then DT may be expressed by a series of cosines of *RQ* and its multiples, the variable coefficients of the series being those of $(a+x)^n$.

23. Suppose both sun and moon in the equator, and prove that the momentary change in the height of the whole tide is proportional to the sine of twice the moon's distance from the place of high water.

24. Let $y = ax^n + bx^{n-1} + cx^{n-2} +$ &c. be the equation to the parabolic curve whose axis is *AE*.



Prove that the continued product of the greatest ordinates will be equal to $\frac{a^{n-1}}{n^n}$ multiplied into $BC^2 \times BD^2 \times CD^2 \times$ &c.

1811.

Morning Problems.—Mr. Turton.

Monday, January 14, 1811.

1. The interior angles of a rectilinear figure are in arithmetic progression; the least angle is 120°, and the common difference 5°. Required the number of sides.

2. Given the radii of two spheres, and the line joining their centers; find, in that line, the position of an eye, to which the apparent surfaces will together be the greatest possible.

3. The weight of a globe in air $= W$, and in water $= w$; find its diameter and specific gravity, having given the specific gravity of water (S) and of air (s).

4. Having given the latitude of the place, the day and hour, also the latitude and longitude of a star; find its altitude and azimuth, the point where its vertical circle cuts the ecliptic, and the angle which they make.

5. Find the ratio of the velocity at the extremity of the latus rectum of an ellipse (the force being in the focus) to the velocity in a circle whose radius is

the distance of the nearer apside from the focus; and shew that, as the excentricity is increased, this ratio approaches to a ratio of equality.

6. Shew that the spaces described by a body, impelled from rest by a finite variable force, are, "ipso motûs initio," in the duplicate ratio of the times.

7. If, to the radius unity, $A =$ the sum of the tangents of any number of arcs; $B =$ the sum of the products of every two of them; $C =$ the sum of the products of every three; and so on: shew that the tangent of the sum of those arcs will be

$$\frac{A - C + E - G + \&c.}{1 - B + D - F + \&c.}$$

8. Shew that the fluent of

$$\frac{d\dot{z}}{z\sqrt{(a + bz^n)}} = \frac{2d}{n\sqrt{a}} \times \text{hyp. log.} \frac{\sqrt{(a + bz^n)} - \sqrt{a}}{\sqrt{(bz^n)}};$$

and find the fluent of

$$\frac{z^{\frac{1}{2}n-1}\dot{z}}{z^{pn} \times (a + bz^n)}, \text{ and of } \frac{\dot{z}}{z^{\frac{1}{2}n+1} \times \sqrt{(a + bz^n)}}.$$

9. Find the value of $a.(a + r).(a + 2r)$ &c. continued to any number of factors.

10. Find the nature and length of the caustic, when the reflecting curve is a circular arc, and the focus of incident rays is in the circumference of the circle.

11. At a given place, at a given hour, and on a given day, required the point of the compass on which a rainbow would appear.

12. Given the latitude of the place, and the day of the year; find the hour at which two stars, whose right ascensions and declinations are known, will be on the same azimuth.

13. Given the perihelion distance of a comet describing a parabola, and the radius of the earth's orbit, here supposed to be circular; compare the time of the comet's moving through 90 degrees of true anomaly with the length of the solar year.

14. Define the center of spontaneous rotation of a system; explain the principle on which that center may be found; and shew that if the system revolve round an axis, passing through that center perpendicular to the plane of revolution, the former point of impact will become the center of percussion.

Monday Afternoon.—Mr. Turton.

FIFTH AND SIXTH CLASSES.

1. The interest of £25. for $3\frac{1}{2}$ years, at simple interest, was found to be £3. 18*s*. 9*d*.; required the rate per cent. per annum.

2. If the first of six magnitudes be to the second as the third to the fourth, and the fifth to the second as the sixth to the fourth; prove that the first and fifth together will be to the second, as the third and sixth together to the fourth.

3. Given $\frac{\sqrt{(a^2+x^2)}}{\sqrt{(a+x)}}$, a minimum; find the value of x.

4. Find the fluent of

$$\frac{d\dot{z}}{\sqrt{(a+bz)}}, \text{ of } \frac{d\dot{z}}{\sqrt{(a+bz^2)}}, \text{ and of } \frac{x\dot{x}}{\sqrt{(ax-x^2)}}.$$

5. Find an expression for the sum of n terms of the series $\frac{1}{5} - \frac{2}{15} + \frac{4}{45}$ &c., that may be applied according as n is an even or an odd number.

6. Shew, that if any momenta be communicated to the parts of a system, its center of gravity will move in the same manner that a body, equal to the sum of the bodies in the system, would move, were it placed in that center, and the same momenta, in the same directions, communicated to it.

7. Compare the time of oscillation in a given cycloid with the time of falling down a vertical line equal to the whole length of the cycloid.

8. Required the equation of which the roots are $\pm\sqrt{(-2)}$, 3, 4.

9. If a body fall through a finite altitude AS, the force varying inversely as the square of the distance, and on AS, a semi-circle ADS be described; prove that the area described by the indefinite radius SD is equal to the area uniformly described in the same time, in a circle whose radius is the half of SA.

10. Given the latitude of the place, and the sun's declination; find the length of the day.

11. Compare the time of descent through any space AS, the force at S varying inversely as the square of the distance, with the periodic time in a circle whose radius is SA.

12. Explain by what means the accelerating forces of bodies are compared; also, by what means their moving forces; and shew that the accelerating force varies as the moving force directly, and the quantity of matter moved inversely.

13. Prove, that if the object placed before a spherical reflector be a straight line, the image is a conic section.

14. Two weights, of which one (P) is known, are connected by a string passing over a fixed pulley; P, in descending from rest through the space s, acquires the velocity a. Find the other weight.

15. Find the variation of the force by which a body describes a parabola, round a center of force in the focus.

16. Find the actual periodic time in a given ellipse, described round a center of force in the focus; supposing that the force at a given distance (d) is to the force of gravity as F to 1.

Mr. Jephson.

THIRD AND FOURTH CLASSES.

1. What is the interest of £115. for $5\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent.?

2. Given $\left.\begin{array}{r} x+y-\dfrac{\sqrt{(x+y)}}{\sqrt{(x-y)}}=\dfrac{6}{x-y} \\ x^2+y^2=\quad 41 \end{array}\right\}$; find x and y; and find x in the equation $a^x+\dfrac{1}{a^x}=b$.

3. Investigate the rule for extracting the square root of a binomial surd, and apply it to determine the square root of $-2n\sqrt{(-m^2)}$.

4. Investigate the fluxional expression for the area of curves, and apply it to find the area that lies between the asymptotes of a common hyperbola.

5. If (a) be an arithmetic, (b) a geometric, (c) a harmonic mean; shew that (a) is greater than (b), and (b) greater than (c).

6. If any quantities, whose differences are inconsiderable in respect to the quantities themselves, be in arithmetical progression, the same quantities are also in geometrical progression.

7. Inscribe, in a triangle, a parallelogram similar to a given parallelogram.

8. Two balls A and B are placed on a billiard-table; in what direction must A, which is perfectly elastic, be struck, that it may hit B after impinging upon two adjacent sides of the table?

9. The specific gravity of a cylinder of known length is greater than that of the fluid in which it is placed; determine the depth of its lower surface.

10. The sine of incidence is to the sine of refraction, out of a denser medium into a rarer, as (n) to (m); give a geometrical construction for determining the greatest possible angle of incidence.

11. Find the fluents of

$$\frac{\dot{x}}{x^3\sqrt{(a^4+x^4)}}, \quad \text{and } \dot{x}\sqrt{(bx-cx^2)}.$$

12. Sum the following series:

$$\frac{1}{2.4.6} + \frac{1}{4.6.8} + \frac{1}{6.8.10} + \text{\&c. in inf.}$$

$$\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \text{\&c. in inf.}$$

13. The angles at the base of an isosceles spherical triangle are equal.

14. Construct a horizontal dial.

15. If the moveable orbit (NEWT. Sect. IX.) be a logarithmic spiral, what is the nature of the curve traced out in fixed space?

16. Prove that one of the roots of the equation $x^3 - qx - r = 0$, when squared, will lie between (q) and $\left(\frac{q}{3}\right)$.

Evening Problems.—Mr. JEPHSON.

1. Shew that $a^n - b^n$ is, and that $a^n + b^n$ is not, divisible by $a - b$. Is $a^n - b^n$ divisible by $a + b$ (n an integer)?

2. The chord is ultimately parallel to the tangent of the middle point of the arc. Required a proof.

3. Prove that the total number of combinations of (n) things is $2^n - 1$; and apply the expression to find the number of different sums that may be formed with a guinea, a half-guinea, a crown, a half-crown, a shilling, and a sixpence.

4. Define similar curves when referred to a point, and shew in what case epicycloids are similar.

5. A lever, at whose extremities P and Q hung by threads, balance each other, is made to revolve about its fulcrum; shew that, if the threads be equal, P and Q describe concentric circles; if unequal, similar segments of circles.

6. Two equal weights balance each other by means of three tacks forming an isosceles triangle, the base of which is horizontal; shew that the whole pressure on the tacks, estimated in the direction in which the weights act, is equal to the sum of the weights.

7. Shew that the pressure of a cylinder, infinite in height upon the earth at rest, equals the weight of another cylinder of the same base, whose length is equal to the radius of the earth.

8. Compare *geometrically* the resistance upon a paraboloid moving in the direction of its axis, with the resistance upon its circumscribing cylinder.

9. If two canals be cut through the earth at rest, then the times of two bodies being attracted through these canals will be equal.

10. $x^y = y^x$; give a geometrical construction for determining two corresponding values of x and y.

11. (1.) Take the fluxion of the two quantities

$$z^{y^z},\ x^{y^z}.$$

(2.) Find the fluents of $\dfrac{\dot{x}}{(a^n+x^n)^{\frac{n+1}{n}}}$,

$$\frac{\dot{x}}{(x-1)^{\frac{3}{2}}.(x+1)^{\frac{1}{2}}},\quad \dot{x}\int \dot{x}\int\frac{\dot{x}}{x}.$$

(3.) Shew that $e^{\int \frac{\dot{\theta}}{\sin. \theta}} = \tan. \left(\frac{\theta}{2}\right)$; (e) being the base of the hyp. logarithms.

12. Sum the following series:

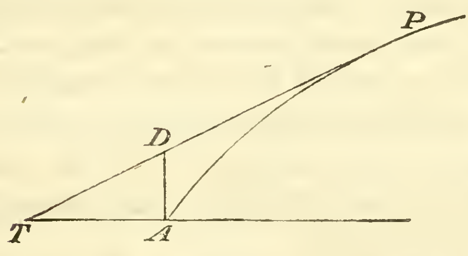
$$\frac{1}{m.(m+r)} + \frac{1}{(m+r).(m+2r)} + \frac{1}{(m+2r).(m+3r)}$$
+ &c. in inf.

$$\frac{5}{1.2.3} + \frac{7}{2.3.4} + \frac{9}{3.4.5} + \text{\&c. to } n \text{ terms.}$$

$$1 + 2.3 + 3.3^2 + 4.3^3 + \text{\&c. to } (n) \text{ terms.}$$

13 Divide a given arc (A) less than a quadrant, into two such parts P and Q, that $(\tan. P)^m \times (\tan. Q)^n$ may be a maximum.

14. PT is a tangent to the curve AP, AD is



perpendicular to the axis; find the nature of the curve when $AT \propto (AD)^m$.

15. A known weight (P) at the extremity of a rod which passes through two small rings fixed in the same vertical line, by its pressure puts a solid inclined plane in motion along an horizontal table; given the weight of the plane, find its elevation so that the

velocity communicated to it in a given time may be a maximum.

16. $\{\text{Cos.} A \pm \sqrt{(-1)}.\sin. A\}^m = \cos. mA \pm \sqrt{(-1)}.\sin. mA$. Required proof.

17. A paraboloid of given dimensions is filled with fluid, and placed with its axis parallel to the horizon; how long will the fluid be in running out of it, through a given orifice in the lowest point of the paraboloid?

18. Let a cylinder begin to move from a horizontal position round one of its ends, which remains fixed upon a fulcrum; compare the pressure on that end at the beginning of the motion, with the whole weight of the cylinder.

19. Two bodies are projected at the same time with velocities v and v' from the two extremities of a vertical line; prove, *geometrically*, that if they meet in the middle point of the line, $v \sim v'$ equals the velocity acquired in the time of meeting.

20. Given the latitude of the place, and the sun's declination; find the time of the day when the hour-angle from noon, and the sun's azimuth from the south, are equal.

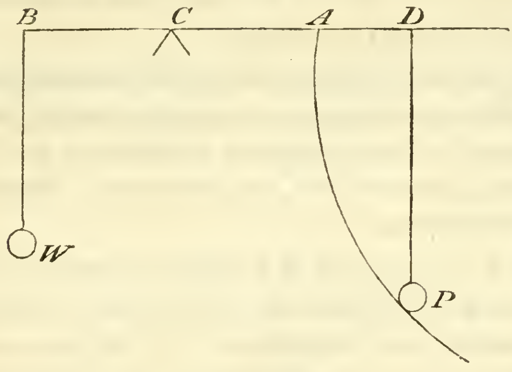
21. Construct the spiral, whose arc is the measure of the ratio between the ordinates which intercept it.

22. If the cycloid described by a body in a resisting medium be formed into a straight line, and ordinates be drawn, which are to the resistances as the length of the pendulum to the weight of the body; then

the area of the curve so traced out is equal to half the length of the cycloid multiplied into the difference between the descent and ascent. NEWT. Vol. II. Prop. xxx. Sect. 6.

23. When a ray of light is incident obliquely upon a spherical reflector, the longitudinal aberration ultimately varies as the versed sine of the arc of the reflector. Required a demonstration.

24. $z^2 = 2ax + x^2$ is the equation to the catenary AP; $AC = CB = (a)$; prove that a weight at B will balance a weight equal to itself made to hang verti-



cally from any point in the axis as D, and pressing upon the catenary; C being the fulcrum of the lever.

25. A cone of given weight and dimensions is placed with its axis horizontal; a known weight (P) is attached to a string, which is wound round its base; find the velocity acquired by P at the end of t''.

Tuesday Morning.—Mr. Jephson.

FIRST AND SECOND CLASSES.

1. Divide (a) into three parts x, y, z, such that $x^m . y^n . z^r$ may be a maximum.

2. From a solid cylinder of given dimensions, cut out a rectangular beam which shall be the strongest possible.

3. Deduce a fluxional expression for the time of emptying vessels through small orifices, and apply it to compare the times of emptying two equal paraboloids; the orifice of the one being in the vertex, and of the other in the base.

4. The focal lengths of a convex and concave lens of different substances, which when united produce images free from colour, are proportional to the dispersing powers of the two mediums. Required a demonstration.

5. Let the weight of a wheel and axle be (w), and let the axis be horizontal; having given a weight (q) applied to the circumference of the axle, and (p) applied to the circumference of the wheel, it is required to find the velocity of the descending weight (p) at the end of t''.

6. Given the sum of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \text{\&c.};$$

find the sum of the series

$$\frac{1}{1^2 . 2^2} + \frac{1}{2^2 . 3^2} + \frac{1}{3^2 . 4^2} + \text{\&c. in inf.}$$

7. Let the roots of the equation $x^n - px^{n-1} + qx^{n-2} - \&c.\ldots - Qx + R = 0$ be (a), (b), (c), (d), &c. It is required to find the value of

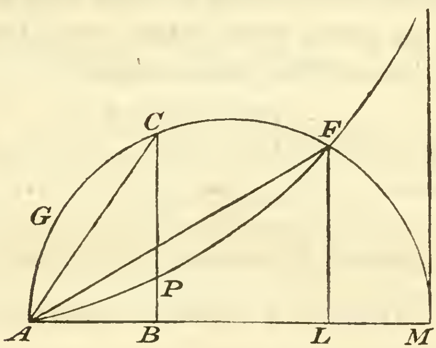
$$\frac{a}{b} + \frac{b}{a} + \frac{a}{c} + \frac{c}{a} + \&c. + \frac{b}{c} + \frac{c}{b} + \&c. + \&c.$$

8. Integrate the fluxional equation

$$\frac{x\dot{y} - y\dot{x}}{x^2 + y^2} = K.\dot{\theta}.$$

9. The error in the altitude of a heavenly body is to the corresponding error in right ascension as the sine of the azimuth to the secant of latitude. Required proof.

10. ACM is a semi-circle; BC, FL are ordinates; the area ABP is always taken equal to the segment ACG; find the equation to the curve traced out by



P; determine the point F, in which the curve cuts the semi-circle, and prove that that part of the area which is without the semi-circle is equal to the rectilineal triangle AFL.

11. A and B begin to fall at the same time from

different points in the same vertical line, and with the velocities acquired move along a horizontal plane, till one overtakes the other; shew that the time of A's descent is equal to the time of B's uniform motion.

12. A bag contains (n) balls, A, B, C, D, &c.; (p) of them are to be taken out; what is the chance of drawing (p) specified balls?

13. The equation to a catenary is $z^2 = 2ax + x^2$; a parabola, whose latus rectum $= (8a)$ is described with the same vertex and axis; shew that any ordinate of the catenary, together with its corresponding arc, is equal to the corresponding arc of the parabola.

14. A chain whose length is (l) is placed along an inclined plane whose height is (n) and length (m), so that one end may coincide with the lowest point of the plane; shew that the whole time of the chain's sliding off the plane is equal to

$$\sqrt{\frac{(m.l)}{(m-n)}} \times \text{hyp. log.} \left\{\frac{m+\sqrt{(m^2-n^2)}}{n}\right\} \times g\text{;} \quad (g = 32\tfrac{1}{6}).$$

Tuesday Afternoon.—Mr. Turton.

THIRD AND FOURTH CLASSES.

1. Explain what is meant by the present worth of money due after a certain time; explain also the principle on which is founded the rule for calculating present worth; and find the present worth of £430. due nine months hence, discount being allowed at $4\frac{1}{2}$ per cent. per annum.

2. Solve the equation $3x^n \sqrt[3]{(x^n)} + \frac{4x^n}{\sqrt[3]{(x^n)}} = 4$.

3. Find the fluent of

$$\frac{x^2 \dot{x}}{\sqrt{(2ax - x^2)}}, \text{ and that of } \frac{\dot{x} \times (2ax - x)^{\frac{3}{2}}}{x}.$$

4. Find the dimensions of the greatest cylinder that can be cut out of a solid formed by the revolution of a curve round its axis, of which the equation is $a^m x^n = y^{m+n}$, and the whole axis $= b$.

5. Given the altitude of an orifice in the side of a vessel filled with fluid, and the distance on the horizontal plane at which the fluid falls; determine, by construction, the altitude of the vessel.

6. If $a =$ the altitude of a conic frustum, and b, c be the radii of its bases, also $p = 3.14159$ &c.; then will the solidity $= \frac{pa}{3} \times (b^2 + bc + c^2)$. Required a proof.

7. The force varying inversely as (dist.)$^{n+1}$, find the area *ABFD* (Newton, Prop. xxxix.) when the ordinate $= M$ at the distance d from the center; also the *fluxion* of the area *ATVME*, the ordinate of this curve at the distance r from the center being $= N$.

8. Having given the latitude of the place, the sun's declination and altitude; find his azimuth, the time of observation, and the angle of position.

9. Having measured the shadow of a tower on the horizontal plane, on a given day, at noon, in a known

latitude; shew how the altitude of the tower may be found.

10. Construct Newton's Telescope, and investigate its magnifying power.

11. Find the variation of the force tending to the focus of an hyperbola, by which the opposite hyperbola may be described.

12. Supposing the attraction of the earth and moon to be as their quantities of matter directly, and the squares of their distances inversely; having given their quantities of matter and their distance, find that point between them, at which a body would be at rest.

13. Make a body oscillate in a given epicycloid.

Mr. Jephson.

FIFTH AND SIXTH CLASSES.

1. Agreed for the carriage of $2\frac{1}{2}$ tons of goods, $2\frac{9}{10}$ mile, for $\frac{3}{40}$ of a guinea; what is that per cwt. for a mile?

2. What is the amount of £120. 10*s*. for $2\frac{1}{2}$ years, at $4\frac{3}{4}$ per cent.?

3. Divide .7584 by 316.; and find the sum of the recurring decimal 5.72323 &c.

4. Solve the following equations:

$$\left.\begin{aligned} x+y+\sqrt{(x+y)} &= 6 \\ x^2+y^2 &= 10 \end{aligned}\right\}$$

$$\left.\frac{x}{x+4}+\frac{4}{\sqrt{(x+4)}}=\frac{21}{x}\right\}$$

5. Prove that if $A \propto B$, $A \pm B \propto B$.

6. Divide $x^3 - nax^2 + na^2x - a^3$ by $x - a$.

7. The number of combinations which (n) things admit of when taken four and four together, is to the number which they admit of when taken two and two, as 15 to 2. Required the number of quantities.

8. Define the sine of an arc; and prove that the sines are the same, drawn from either extremity of the arc.

9. Given the center of a circle; find its diameter by means of the compasses alone.

10. Take the fluxion of $(a^2 + x^2) . \sqrt{(a^2 - x^2)}$; and find the $\int \frac{a^2 \dot{x}}{b^4 + c^2 x^2}$.

11. $\frac{x^{n+1}}{2a^n} + \frac{a^n}{2x^{n-1}} =$ a maximum; find x.

12. The sum of (n) terms of the series 1, 3, 5, 7, &c. is to the sum of $(n - 1)$ terms of the series 2, 4, 6, &c. as (n) to $(n - 1)$. Required proof.

13. Let a perfectly hard body (A) impinge upon another (B) four times as great, and at rest, with a known velocity; find B's velocity after impact.

14. An eye is placed in the principal focus of a concave spherical reflector; compare the apparent magnitudes of the object and image, when the former is situated half way between the focus and surface.

15. An inverted cone is filled with a fluid; deter-

mine at what distance from the vertex a horizontal section will sustain the greatest pressure.

16. When the center of force is without the circle, find its variation in terms of the variable distance.

17. Let a body begin to fall from an infinite distance, force varying as $\frac{1}{(\text{dist.})^2}$: shew that its velocity at any point of its descent is equal to the velocity that it would acquire through the remaining distance, force at that point being continued constant.

Evening Problems.—Mr. TURTON.

1. Define similar curves when referred to their axes; and prove that all parabolas are similar curves.

2. A given rectangular parallelogram is immersed vertically in a fluid, with one side coincident with the surface. From one of its angles, it is required to draw a straight line to the base, so that the pressures on the parts into which the parallelogram is divided may be in a given ratio.

3. Find that point, in the periphery of an ellipse, from which a body must fall, towards the center of force in the focus, through the greatest or least space to acquire the velocity in the curve, at the point from which it fell; and shew, from the fluxional

equation, whether the point determined gives the space a maximum or minimum.

4. Construct GREGORIE's Telescope, and investigate its magnifying power.

5. The times of falling, from different altitudes, into the same center of force, vary as the n^{th} powers of those altitudes. Required the variation of the force.

6. Suppose a person to stand before a vertical plane mirror, at any distance from it; given the altitude of the eye above the bottom of the mirror, find the part of the body that will be seen; and shew that exactly the same part will be visible at all distances from the mirror.

7. If parallel rays be incident on a sphere of given refracting power, find that ray, of which when produced, the part included by the sphere will be to the part included of the refracted ray, in a given ratio.

8. On a given day, at a given hour, and in a given latitude, it is required to find the length and direction of the shadow of a stick of given length, inclined to the horizon at a given angle, and in a given direction.

9. Resolve $\frac{1}{1-2lz+z^{2n}}$ into quadratic factors, when l is less than unity.

10. In the expansion of $(a+b+c+d+\&c.)^m$, investigate the coefficient of the term involving the literal product $a^p b^q c^r d^s$ &c.

11. Given $a\dot{x}+(bx+cy).\dot{x}=d\dot{y}+(bx+cy).n\dot{x}$; find the relation between x and y.

12. Sum the following series: viz.

$$\frac{3}{1^2.2^2}+\frac{5}{2^2.3^2}+\frac{7}{3^2.4^2}+\text{\&c. to } n \text{ terms,}$$

and ad inf.

$$\frac{1}{1.2.3.4.5}+\frac{4}{4.5.6.7.8}+\frac{9}{7.8.9.10.11}+\text{\&c.}$$

to n terms, and ad inf.

13. Find the fluents of

$$\dot{x}\int\frac{\dot{x}}{1-x^3}, \text{ and of } \frac{z^{2n-1}\dot{z}}{(g+hz^n)\sqrt{(e+fz^n)}}.$$

14. Given the latitude and longitude of a fixed star; required the angle which the direction of the earth's way makes with a line drawn from the earth to the star, on a given day.

15. Find the time of the year at which a star, whose right ascension and declination are known, rises with the sun, to an observer in a given latitude.

16. Shew that the force of resistance on a sphere, moving in a fluid, with a given velocity, is to the force that would destroy the sphere's whole motion, in the time in which it would uniformly describe $\frac{8}{3}$ of its diameter, as the density of the fluid to that of the sphere.

17. Find the value of the disturbing force of S upon P, in the case of the three bodies (Sect. XI.); and deduce the mean quantity of that force, during

one revolution of P round T; supposing that $M=$ the force of S at the distance r.

18. If the density of a fluid be proportional to the compressing force, and its particles be attracted by a force varying inversely as the distance from the center; shew that, distances from the center being taken in geometric progression, the corresponding densities will be in geometric progression.

19. Investigate COTES's method of determining the length of an arc of the meridian, on the planisphere.

20. Shew that, in any position of the moon's nodes, the mean horary motion of the nodes, in one synodic revolution of the moon, is equal to half their horary motion when the moon is in syzygies.

21. Having given two distances of a comet, in its parabolic orbit, from the sun, and the angle included; deduce this proportion for determining the perihelion (which is here supposed to lie between those distances.)—The sum of the square roots of the distances is to the difference as the co-tangent of the semi-sum of half the true anomalies to the tangent of the semi-difference of the same.

22. Apply NEWTON's general proposition, in the 8th Section, to the case of COTES's three last spirals; point out the circumstances that determine the spiral; and deduce his construction of the elliptic spiral.

23. The major and minor axes of an ellipse are

given; and a body begins to descend, from the extremity of the minor axis, towards the center of force in the focus, with the velocity in the curve at that point. Compare the time of descent to the focus with the time of revolving in the curve, from the same point to the nearer apside.

24. Suppose a sphere to move in a resisting medium; it is required to cut off a segment, by a plane perpendicular to the direction of its motion, so that the resistance on the remaining frustum may be three fourths of that on the end of a cylinder circumscribing the sphere.

1812.

Monday Morning—Mr. Hustler.

Monday, January 15, 1812.

FIRST AND SECOND CLASSES.

1. If two straight lines are parallel, the common section of any two planes passing through them is parallel to either.

2. The length of the tropical year being $365^d\ 5^h\ 48'\ 48''$, explain the reason why three out of four hundredth years are *not* leap-years.

3. Shew the method of discovering whether an equation has any equal roots; and apply it to the solution of the equation,

$$x^5 - 13x^4 + 67x^3 - 171x^2 + 216x - 108 = 0.$$

4. Determine the evolute of the common parabola.

5. Compare the resistance to a given cylinder moving in a fluid in a direction perpendicular to its axis with the resistance to the same cylinder moving with the same velocity in the direction of its axis.

6. At a given hour, on a given day, at a given place, to determine the latitude and longitude of the mid-heaven.

7. If S be the point of suspension of an oscillating body, G the center of gravity, O the center of oscillation; and from center G with radii GS, GO, circles be described in the plane of oscillation; then the axis of suspension being removed to any point in either circumference, the pendulum will oscillate in the same time as before.

8. Find the increment of the number whose hyperbolic logarithm is x.

9. Shew that the fluent of

$$\frac{\dot{z}}{\text{sec. } z \,.\, \text{cosec. } z} = \frac{1}{4} \text{ vers. sin. } 2z.$$

10. A paraboloid, laid upon a horizontal plane, rests with its axis inclined to the horizon at 30°. Compare the length of the axis with the latus rectum.

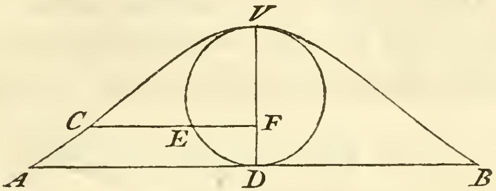
11. If a body be projected at an angle with the velocity acquired in falling from an infinite distance, force varying as $\frac{1}{(\text{dist.})^n}$, compare the chord of curvature of the orbit at the point of projection, with the distance.

12. Sum the following series:

$$\left.\begin{array}{l} \frac{5}{1.4} - \frac{7}{2.5} + \frac{9}{3.6}, \text{ \&c.} \\ \frac{1}{1.3.4} + \frac{4}{2.4.5} + \frac{7}{3.5.6}, \text{ \&c.} \end{array}\right\} \text{to } n \text{ terms and in inf.}$$

$$1 - \frac{1}{3} + \frac{1.2}{3.4} - \frac{1.2.3}{3.4.5} + \frac{1.2.3.4}{3.4.5.6}, \text{ \&c. to inf.}$$

13. *AVB* is the *trochoid* of Newton, in the sixth Section; *VD* its axis; *CEF* an ordinate parallel to



the base *AB*, cutting the curve in *C*, and the circle on the axis in *E*. Shew that the arc *VE* is to the line *CE* in a constant ratio.

14. If several circles be described, the force tending to a common point in all their circumferences, the periodic times are as the cubes of the radii. Required a proof.

Monday Afternoon.—Mr. Hustler.

FIFTH AND SIXTH CLASSES.

1. Prove that in the multiplication of decimals, there are as many decimal places in the product as in the multiplier and multiplicand together.

2. Find two numbers, such that if $\frac{2}{3}$ of the less be added to $\frac{1}{4}$ of the greater, the sum will be 7; but if $\frac{1}{3}$ of the greater be taken from the less, the remainder will be 2.

3. Extract the square root of $6\sqrt{(-2)}-3$.

4. If a perpendicular be let fall from the vertex of any triangle upon the base, the rectangle by the sides

of the triangle = the rectangle by the perpendicular and the diameter of the circumscribed circle.

5. Having given the sine of an angle, it is required to find the cosine of twice the angle.

6. Sum the series, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3, &c. to 13 terms.

also, $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7}$ &c. to n terms.

7. Draw a meridian line, and shew the method of finding the sun's meridian altitude by experiment.

8. If a body revolve in a circle, the force being in a point of the circumference; required the law of the force.

9. The square of any odd number increased by 3 is divisible by 4.

10. Shew that the angular velocity in different conic sections about the same focus $\propto \frac{\sqrt{(\text{latus rectum})}}{(\text{dist.})^2}$.

11. Find the fluxion of

$(a + bx^{\frac{3}{2}} + cx^{\frac{2}{3}})^{\frac{1}{2}}$ and of hyp. log. of $\frac{\sqrt{(a^2 + x^2)}}{\sqrt{(a^2 - x^2)}}$;

also the fluent of $\frac{x\dot{x} + \dot{x}}{x\sqrt{(a^2 + x^2)}}$.

12. Prove that all the images of an object placed between two plane mirrors inclined at a given angle will lie in the circumference of a certain circle.

13. Required the amount of £56. 13*s*. 4*d*. put out to simple interest for 5 years and 4 months at 6 per cent. per annum.

14. Prove that the time down different inclined planes $\propto \frac{\text{length}}{\sqrt{(\text{height})}}$.

15. If a body float on a fluid, the part immersed : whole body :: specific gravity of body : specific gravity of fluid.

16. Prove that all rays coming parallel to the axis of a parabolic reflector will converge accurately to the focus.

17. In the equation $x^3 - 10x^2 + 27x - 18 = 0$, the greatest root is double of the second and the second treble of the third. Find all the roots.

Monday Afternoon.—Mr. TURTON.

THIRD AND FOURTH CLASSES.

1. Given the series arising from the expansion of $(a+b)^m$; determine the n^{th} term.

2. Find the equation whose roots are $1 \pm \sqrt{(-2)}$, $2 \pm \sqrt{(-3)}$.

3. The roots of the equation $x^3 - px^2 + qx - r = 0$ are a, b, c; find the equation whose roots are ab, ac, bc.

4. Find the sum of n terms of the series $1^2 + 3^2 + 5^2 + \&c.$

5. Find the fluents of the following quantities:

$$\frac{x^5 \dot{x}}{\sqrt{(a^2 - x^2)}}, \quad \frac{\dot{x}}{x^4 \sqrt{(a^2 + x^2)}}.$$

6. Determine, by geometrical construction, that point in a cycloid at which the velocity of an oscillating body is half the greatest velocity.

7. Shew that, at any point in an ellipse, the increase of the focal distance is to that of the perpendicular on the tangent as $CD \times HP$ to $AC \times CB$.

8. A hemisphere, resting on a fluid with its vertex downwards, has two thirds of its axis immersed; compare the specific gravities of the fluid and solid.

9. If a=the altitude of a parabolic frustum, and b, c be the radii of its bases, also p=3.14159; then will the solidity $= \frac{pa}{2} \times (b^2+c^2)$: required a proof.

10. If the reflecting curve be the arc of a given cycloid, the rays being incident parallel to the axis; required the nature and length of the caustic.

11. Determine the points of the compass on which the sun will rise and set, to an observer at a given place, on a given day of the year.

12. Given the earth's radius and the space fallen through in one second at its surface; also the periodic time of the moon; required the moon's distance, gravity varying inversely as the square of the distance from the earth's center.

13. A body descending from rest in a fluid acquires a velocity (a) in falling through the space

(s). Compare the specific gravities of the fluid and body, the resistance of the fluid being neglected.

14. Compare the time of moving through the apside, from one extremity of the latus rectum to the other, in different parabolas, round different centers of force in the foci.

Monday Evening.—Mr. Turton.

1. Prove that, if from any point in the directrix of a parabola two tangents be drawn to the curve, those tangents will be at right angles to each other.

2. In treatises on mensuration, the expression $\frac{8 \text{ chord } \frac{1}{2} A - \text{chord } A}{3}$ is given as an approximate value of the arc (A) of a circle: investigate the truth of this approximation.

3. Give Cotes's constructions for determining the orbits described by bodies acted upon by forces varying inversely as the square of the distance.

4. There is a point in the circumference of a circle from which the circumference is suspended. Shew that, if two equal weights be fixed at any points whatever in the circumference, equally distant from the point of suspension, and be made to vibrate in the plane of the circle, the time of oscillation will be equal to that of a pendulum whose

length is the diameter of the circle.—(The circumference is devoid of gravity.)

5. Prove that the time of falling from rest from any point (P) in a parabola to the center of force the focus (S) is to the time of moving in the curve from that point to the vertex (A) as $\frac{3p}{4} \times SP^{\frac{3}{2}}$ to $(SP + 2SA)\sqrt{(SP - SA)}$ $(p = 3.14159)$.

6. Solve the fluxional equation $\dot{y} + Py\dot{x} = Q\dot{x}$, where P and Q are functions of x.

7. Shew that, between the values $x = 0$, and $x = 1$,

$$\int \cdot \frac{x^{2n+1}\dot{x}}{\sqrt{(1-x^2)}} = \frac{2.4.6.\ldots\ldots 2n}{3.5.7.\ldots\ldots(2n+1)},$$

and $$\int \cdot \frac{x^{2n}\dot{x}}{\sqrt{(1-x^2)}} = \frac{1.3.5.\ldots\ldots(2n-1)}{2.4.6.\ldots\ldots 2n} \times \frac{\pi}{2}$$

(π = half the circumference of a circle to the radius 1): and from these fluents deduce WALLIS's expression for the circumference of a circle to the same radius; viz. $4 \times \frac{2^2.4^2.6^2.\text{ ad inf.}}{1\ .3^2.5^2.\text{ ad inf.}}$.

8. If a straight line of given length pass through a fixed point, and one end move along a straight line given in position, construct the curve which will be the locus of the other extremity.

9. Shew in what case the cycloid *within* the globe (in the 10th Section) becomes a straight line: and find, from the requisite data, the time of oscillating in that line.

10. Shew that, when in any curve the velocity is less than that in a circle at the same distance, the angle between the radius vector and *direction of the body's motion* continually diminishes: and that when the velocity is greater than that in a circle the said angle continually increases.

11. Having given two altitudes of a star, whose declination is known, and the times of observation, on a given day, find its right ascension and the latitude of the place.

12. Given the latitude and longitude of three places on the earth's surface; find the latitude and longitude of one equally distant from them all.

13. Given two distances from the pole of a logarithmic spiral, and the angle between them—shew how the spiral may be constructed.

14. Determine the *form* of a vessel of given altitude which being filled with a fluid and a given orifice being opened in the bottom, the velocity of the descending surface will vary as the n^{th} power of the altitude of the surface above the orifice: and find the *content* of the vessel when the surface begins to descend with a given velocity.

15. Shew how to determine, from three observations, the direction in which a comet is moving, supposing the motion to be uniform and rectilinear.

16. If a body be projected from the earth's surface in a direction making an angle of 45° with

the horizon, and with the velocity of a body revolving in a circle at the surface; required the point at which it will reach the earth again, and the time of motion.

17. Explain the formation of figurate numbers, and shew that if the figurate numbers of any order be divided by the corresponding numbers of the next order, the sum will be infinite; but that if they be divided by those of the next order but one, the sum will be finite.

18. Determine the conic frustum, of a given base and altitude, on which, when moving in the direction of its axis, with its less end foremost, the resistance will be less than that on any other frustum of the same base and altitude.

19. If a body whirled round by a string, describe a circle in a vertical plane, shew that the string cannot retain the body in the circle, unless it can support six times the weight of the body.

20. Prove that if the center of the generating circle of a cycloid move with half the velocity which a body would acquire in falling through its diameter, the describing point will move in the same manner as a body oscillating in the cycloid.

21. Investigate TAYLOR's theorem: and shew, from the theorem, that when the ordinate of a curve is either a maximum or minimum, the *first* fluxion vanishes; and that the maximum *or* minimum will be determined by the second fluxion being negative or positive.

22. A triangular prism, with three unequal sides, rests on a fluid with one angle immersed; having given the point in one side through which the surface of the fluid will pass, find the position of the body.

23. A piston, closely fitting a vertical tube, will, by compressing the air as it descends by its own weight, rest at the altitude $\frac{a}{n}$ (a being the whole altitude). Now suppose the piston to be forced down to the altitude $\frac{a}{pn}$ and there left to the action of the compressed air; find the velocity at any point of its ascent; friction being neglected.

24. A prismatic vessel, of given dimensions, with its sides vertical, is filled with a fluid: there are two given and equal orifices, one at the bottom, the other bisecting the altitude; required the time of emptying the upper half, supposing both orifices to be opened at the same instant.

Tuesday Morning.—Mr. TURTON.

1. Shew that in addition, subtraction, multiplication and division of quantities of the form $a \pm b\sqrt{(-1)}$, and also in the involution of that quantity, the results will always be of the form $A \pm B\sqrt{(-1)}$.

2. The roots of the recurring equation $x^4+px^2+1=0$ must be of the form a, b, $\frac{1}{a}$, $\frac{1}{b}$: exhibit them in that form.

3. Find the fluent of

$$\frac{d\dot{z}\times(a+bz^n)^{\frac{5}{2}}}{z}, \text{ and of } \frac{d\dot{z}}{z\times(a+bz^n)^{\frac{5}{2}}}.$$

4. Compare geometrically the resistance on the curve of a cycloid, moving in the direction of its axis, with that which would oppose the base.

5. Find, from the requisite data, the actual velocity, acquired in falling through the space AD, in terms of the area $ABFD$. (NEWTON, Prop. 39.)

6. Give a definition of finite curvature; and determine the nature of the curvature at a point (P) of a curve at which $\frac{QP^3}{QR}$ approaches to a given area (A^2) as its limit.

7. Shew that the expression for the force in the moveable orbit $\left(\frac{F^2}{A^2}+\frac{RG^2-RF^2}{A^3}\right)$, when applied to orbits nearly circular, continually approaches to $A^{\frac{F^2}{G^2}-3}$ as its limit.

8. Having given the latitude and longitude of a star, find its angle of position.

9. Find the annual variation of the right ascen-

sion and declination of a star arising from the precession of the equinoxes.

10. If A be the arc of a circle whose radius is unity, prove that

$$\log. \cos. A = -M\left(\frac{A^2}{2} + \frac{A^4}{3.4} + \frac{A^6}{5.9} + \&c.\right)$$

11. A parabola revolves round its axis, which is vertical, in a given time, and the angular motion will just prevent a body, at any point of the curve, from descending. Required the parameter of the parabola.

12. Prove that, if different reciprocal spirals be described round the same center of force, the areas described in the same time, in those curves, will be equal.

13. Investigate the n^{th} integral of xv.

14. Two weights P and W are connected by a string passing over a fixed pulley; find the velocity of P at any point of its descent from rest, and also the time of descent; the weight of the string and pulley being considered.

Tuesday Afternoon.—Mr. TURTON.

FIFTH AND SIXTH CLASSES.

1. Given $x+y=a$, and $x^3+y^3=b^3$; find x and y.

2. Divide 1 by $2a-x$.

3. Find the number from the m^{th} root of which, if the n^{th} root be subtracted, the remainder will be the greatest possible.

4. Shew that the tangent of 45° is increasing twice as fast as the corresponding arc.

5. If a body be kept at rest by three forces, and lines be drawn equally inclined to the directions in which they act, forming a triangle, shew that the sides of this triangle will represent the quantities of the forces.

6. Find the fluent of

$$\frac{bx\dot{x}}{(x-a).(x+a)}, \text{ and of } \frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}.$$

7. Given the three sides of a plane triangle; shew how the angles may be found.

8. Given the velocity and direction of projection; find the range on a given inclined plane passing through the point of projection.

9. In a given circle inscribe an equilateral triangle; and shew that the square of the side of the triangle is triple the square of the radius of the circle.

10. Given the longest diagonal of a rhombus $=a$, and one angle $=60^0$. Required its area.

11. Find the position of a straight line down which the time of falling will be twice the time down the same line when perpendicular to the horizon.

12. Two given weights, A and B, are suspended at the extremities of a uniform straight lever of given length (a) whose weight is W; required the

distance of the fulcrum from one end in case of equilibrium.

13. If the force vary inversely as the cube of the distance, prove that the velocity will vary as the tangent, and the time as the sine of a circular arc, whose radius is the greatest distance, and versed sine the space fallen through.

14. Find how far a body must fall internally, towards the center of force in the focus of an ellipse, to acquire the velocity in the curve, at the point from which it fell.

15. If a body be projected with a given velocity and be acted upon by a given uniformly accelerating force; investigate the principle on which the space described in a given time is determined.

16. Shew that the periodic times in ellipses, described round the same focus, are in the sesquiplicate ratio of their major axes.

17. A given rectangular parallelogram is immersed vertically in a fluid with one side coincident with the surface. Divide it, by a line parallel to the surface, into two parts that will be equally pressed.

Tuesday Afternoon.—Mr. Hustler.

THIRD AND FOURTH CLASSES.

1. Prove that an harmonic mean is less than a geometric.

2. Find the amount of an annuity of £140. payable quarterly, for three years, simple interest being allowed at 5 per cent.

3. In the common parabola, the normal is a mean proportional between the latus rectum and the focal distance. Required a proof.

4. Prove that the chord of 120^0 = tangent of 60^0.

5. A body projected at an angle of 60^0 hits a mark at the distance of 300 feet upon an inclined plane whose elevation is 30^0. Required the velocity of projection, the greatest height above the plane, and the time of flight.

6. Shew that the line joining the moon's cusps is perpendicular to the plane passing through the centers of the Sun, Earth, and Moon.

7. If from the center of an ellipse, with radius equal to the line joining the extremities of the axes, a circle be described; a body let fall from any point in its circumference towards the center, will acquire at the point where it meets the ellipse the velocity which a body revolving in the ellipse about the center would have at the same point.

8. The equations $x^3 - 3x^2 + 11x - 9 = 0$, and $x^3 - 5x^2 + 11x - 7 = 0$ have a common root; find all the roots.

9. If a cylindrical vessel filled with a fluid and placed perpendicular to the horizon, empty itself

through an orifice at the bottom, shew that the velocity of the descending surface will be uniformly retarded.

10. Prove according to NEWTON'S 39th Prop., that if the force vary as the distance, the velocity in a straight line is as the sine of a circular arc whose radius is the whole distance, and versed sine the space described.

11. Find the fluents of

$$\frac{x^3 \dot{x}}{1+x^2}, \text{ of } x^6 \dot{x} \sqrt{(a^2+x^2)}, \text{ and of } \frac{z^{\frac{1}{2}n-1} \dot{z}}{a^n+z^n}.$$

12. If the angle between the apsides in an orbit nearly circular be 60°, how does the force vary?

13. If a force varying as the distance tend to the center of a globe, the times of oscillation in all arcs of the hypocycloid are equal.

Tuesday Evening.—Mr. HUSTLER.

1. Two bills, one for £.a payable (b) months after date, the other for £.c payable (d) months after date, are presented at a banker's, who advances £.P for them. Required the rate of simple interest.

2. The excess of the sine above the versed sine is greater for 45° than for any other arc less than a quadrant.

3. Find the center of gravity of a portion of a given paraboloid, cut off by any plane.

4. If the arc of a common cycloid in which a pendulum oscillates be divided into four equal arcs, the time through the first quarter = $\frac{1}{3}$ of the whole time of oscillation.

5. In the reciprocal spiral, the tangent of the angle made by the radius vector with the curve varies inversely as the distance. Required a proof.

6. With two dice, compare the chance of throwing the number 7 in one trial with the chance of throwing it twice in three trials.

7. If a body revolve in an ellipse, the force being in one focus, the angular velocity about the other focus is not *accurately* equal to the mean angular velocity, except at four points. Determine those points.

8. In the stereographic projection of the sphere, a great circle not passing through the pole is projected into a circle whose radius is the secant of inclination to the plane of the projection; and the distance of its center from the center of the sphere is the tangent of the same angle.

9. Sum the series

$$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} \text{ \&c. in inf.}$$

10. The length and distance of a straight object placed before a concave spherical reflector being given, it is required to determine the axes of the image.

11. If several bodies be projected from different distances towards the center, force varying as $\frac{1}{(\text{dist.})^2}$, with the velocities acquired in falling from infinity at those distances respectively; shew, according to Newton's 39th Prop., that the times of falling into the center are in the sesquiplicate ratio of the initial distances.

12. The roots of the equation $x^n - px^{n-1} + qx^{n-2} \ldots - Qx + R = 0$, being a, b, c, &c. shew that $\frac{a^2}{b} + \frac{a^2}{c} + \&c. + \frac{b^2}{a} + \frac{b^2}{c} + \&c. + \frac{c^2}{a} + \frac{c^2}{b} +$ &c. $= \{p^2 - 2q\} \cdot \frac{Q}{R} - p$.

13. At a given place, to determine the day when a given star is due south at sun-rise.

14. A sphere acted upon by gravity is projected downwards in a medium with a velocity greater than the greatest acquirable velocity in the medium; determine the velocity after any space has been described, and the limit when the space is infinite.

15. If an ordinate parallel to the base of a common cycloid cut the curve in P and the generating circle in Q, and tangents PT, QT to the cycloid and circle be drawn, meeting in T, the locus of all the points T is the involute of the generating circle.

16. Find the fluents of

$\frac{\dot{x}}{1+x^n}$, (n) being an odd number, of $\frac{\dot{x}\sqrt{(a^2-x^2)}}{x^6}$,

and of $\frac{-\dot{x}}{\sqrt{\left(\text{hyp. log. } \frac{a}{x}\right)}}$.

17. If the rhumb-line be always inclined to the meridians at 60^0, its length from the equator to the pole = half a great circle of the sphere.

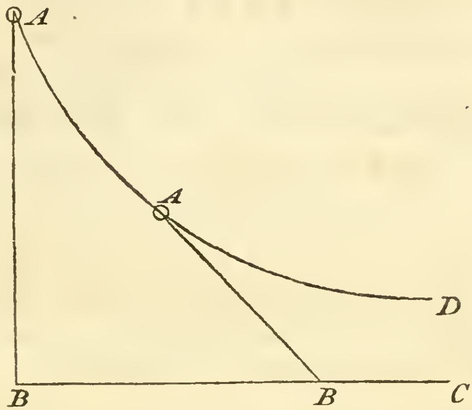
18. Determine the curve of aberration in a plane parallel to the ecliptic, if the orbit of the earth were a circle, and the sun in the circumference.

19. A cube is bisected diagonally by a plane, and one half being filled with a fluid is placed, vertex downwards, with the bisecting plane parallel to the horizon. Find the time of emptying through a small orifice made at the lowest part.

20. Supposing gravity to be constant and perpendicular to the horizon, and that the resistance of a medium varies as density of medium $\times$ $(\text{vel.})^2$ of the body; required the law of density, so that a body may describe a given circle. (Newton, Vol. II. Prop. x.)

21. A weight A attached to a string AB, being laid on a *horizontal* plane $ABCD$, the extremity B of the string is moved along a line BC which is at first perpendicular to BA, and the weight A traces out a curve AD on the plane. Shew that the

surface of the solid made by the revolution of this



curve indefinite in extent about BC = eight times its area.

22. Find the length of the tide-day at new and full moon, and shew that the tide-day is at all times greater than the solar-day.

23. The first term of an arithmetic series is (a), the last term (l) the common difference (d): also S_1, S_2, S_3 ...... S_{m-1} are the sums of the first, second, third, $(m-1)^{\text{th}}$ powers of the terms; shew that $(l+d)^m - a^m = m.d.S_{m-1} + m.\frac{m-1}{2}d^2.S_{m-2} + m.\frac{m-1}{2}.\frac{m-2}{3}.d^3.S_{m-3}$ +&c.

24. A great circle revolves about the axis of a sphere with an uniform angular velocity quadruple of that which a point setting off from the pole advances along it. Find the two surfaces into which the motion of the point divides the hemisphere.

1813.

Monday Morning.—Mr. Macfarlan.

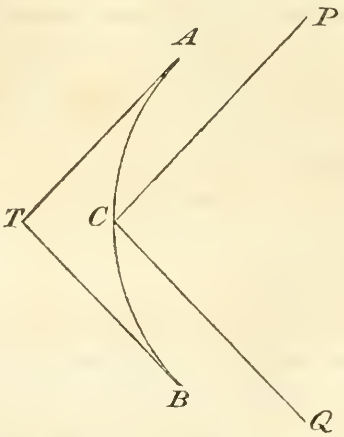
MONDAY, JANUARY 18, 1813.

FIRST AND SECOND CLASSES.

1. Sum the series, cos. A + cos. $2A$ + cos. $3A$. . . + cos. nA where nA = whole circumference. Also the series $\frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5}$; &c. ad inf.

2. Find the fluent $\frac{\dot{x}}{\sqrt{(1+x^2)} - \sqrt{(1-x^2)}}$; and prove that the fluent of $x^x \dot{x}$ between the values of $x=0$, and $x=1$, is $\frac{1}{1} - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} +$ &c.

3. AB is a spherical reflecting arc; C the middle



point; AT and BT tangents from the extremities, to which CP and CQ are drawn parallel; it is required in these lines to find two points, P and Q, so that all rays proceeding from P and incident at A or B, may after reflection converge to Q.

4. Find the quantity of refraction by the circumpolar stars, (Boscovich's method) the refraction being supposed to vary as the tangent of the apparent zenith distance.

5. The angle contained by the two equal sides of an isosceles spherical triangle is greater than the angle contained by the chords of the same sides.

6. Find the value of the fraction $\frac{a^x - b^x}{x}$ when $x=0$, and of $(1-x)$ tan. $\frac{\pi x}{2}$ when $x=1$, π being = semi-circumference of a circle rad. = 1.

7. If a straight line be drawn through the center of gravity of a triangle to meet two sides and the third side produced; the rectangle under the segments of this line measured from the center of gravity on one side of it is equal to the sum of the rectangles under the same two segments and the segment on the other side of the center of gravity.

8. Determine the position in which a lever of given length and uniform thickness will rest between two given inclined planes.

9. If the resistance vary as the velocity, and the force of gravity be constant, the times of describing

all chords of a circle terminating in the extremity of a vertical diameter are equal.

10. In moving from the equator to the pole, the increase of a degree of latitude varies as the square of the sine of latitude.

11. A body revolves in a circle, the center of force being in the periphery. Investigate the nature of the curve traced out by the extremity of the perpendicular upon the tangent; find its area and length, and the value of the greatest ordinate.

12. The density of a lever of given length varies as the n^{th} power of the distance from one extremity, by which it is suspended. A given weight (P) attached to the other end, and acting perpendicularly by means of a pulley, keeps the lever horizontal. The lever (when P is removed) would vibrate m times in t''. Required the weight of the lever, and the index (n).

13. A body, urged by a force varying inversely as the square of the distance, describes from rest a given straight line, while the line itself revolving uniformly performs one complete revolution. Required the area described.

14. The perimeter of an equilateral triangle inscribed in a circle is greater than the perimeter of any other isosceles triangle inscribed in the same circle.

Monday Afternoon.—Mr. MACFARLAN.

FIFTH AND SIXTH CLASSES.

1. Reduce $\dfrac{x^8+x^6y^2+x^2y+y^3}{x^4-y^4}$ to its lowest terms; and prove the Rule for finding the greatest common measure of two quantities a and b.

2. Extract the square root of 315.271, and show generally that if there be n figures in the root there cannot be more than $2n$ nor less than $(2n-1)$ figures in the number whose root is to be extracted.

3. Prove that the hypothenuse of a right-angled triangle is less than the sum of the two sides by the diameter of the inscribed circle.

4. Given the tangents of two arcs; find the tangents of their sum and difference.

5. When the force varies inversely as the square of the dist. the periodic times in ellipses vary as $\dfrac{(\text{axis major})^{\frac{3}{2}}}{\sqrt{(\text{abs. force})}}$.

6. Solve the following equations:

$$\left.\begin{aligned} xz &= y^2 \\ x+y+z &= 21 \\ x^2+y^2+z^2 &= 189 \end{aligned}\right\}$$

$$\sqrt{(5+x)}+\sqrt{x}=\frac{15}{\sqrt{(5+x)}}$$

$$\left.\begin{aligned} x^2+y^2 &= 34 \\ x^2-xy &= 10 \end{aligned}\right\}$$

7. Prove that a geometric mean between two quantities is a mean proportional between an arith-

metic and an harmonic mean between the same two quantities, and show of these three mean terms which is the greatest.

8. Extract the square root of $a^2 - b^2$ to four terms by the binomial theorem.

9. Find the fluxions of $(a+x) \times \sqrt{(a-x)}$; of a^x; and the fluent of $\frac{2a\dot{x}}{a^2 - x^2}$.

10. A body is projected from the bottom of a given inclined plane with a given velocity; find the direction when the range will be a maximum.

11. Find the focal length of a glass sphere.

12. The specific gravity of gold and silver being (a) and (b) and of their compound (c). Find the ratio of the quantities of the gold and silver in the mixture.

13. Given the latitude of the place and the sun's altitude at six o'clock; find the time of the year; and give the proportions for solving the spherical triangle.

14. Construct the supplemental triangle, and prove its properties.

15. In the parabola the rectangle under the principal latus rectum and the abscissa is equal to the square of a semi-ordinate to the axis.

16. Prove the rule for the extraction of the square root of a binomial surd, and apply the expression to $7-2\sqrt{3}$.

17. In a triangle whose sides are (a) and (b), and the included angle $\frac{4}{3}$ of a right angle, the square of the base $= \dfrac{a^3 \sim b^3}{a \sim b}$.

Monday Afternoon.—Mr. Jephson.

THIRD AND FOURTH CLASSES.

1. Investigate the rule for extracting the square root of a binomial surd, and apply it to find the root of $2 + 2\sqrt{(1 - a^2)}$.

2. The true zenith distance of the polar star when it first passes the meridian is $46^0. 50'. 40''. 75'''$. and at the second passage is $50^0. 25'. 50''. 30'''$. Required the latitude of the place.

3. If any number of circles be drawn through two given points A and B cutting a given circle, the lines which join the points of intersection shall all meet AB produced in the same point.

4. In a system of (n) equal pullies, each hanging by a separate string, and the strings parallel, having given P and W and the weight of one pulley, find (n) when there is an equilibrium.

5. If a vessel be filled with fluid, the pressure on any part : the weight of the fluid :: area of that part $\times$ the depth of its centre of gravity : solid content of the fluid.

6. If $p + 1 : 1$ be the ratio of the tangents of two

angles, and $m : n$ the ratio of their sines, shew that $p+1 : 1$ is always greater than $m : n$.

7. Prove that

$$f. \frac{\dot{x}}{\sqrt{(x^2+2ax)}} = \tfrac{1}{2} \text{ hyp. log.} \left\{ \frac{\sqrt{x}+\sqrt{(x+2a)}}{\sqrt{2}} \right\},$$

and that

$$f. \frac{m\dot{z}}{a+bz^2} = \frac{m}{\sqrt{(ab)}} \text{. arc. tang.} = z\sqrt{\frac{b}{a}} \text{. (rad. 1).}$$

8. Sum the series $2^2+4^2+6^2+8^2+$ &c. to (n) terms; and shew that the series $1^3+2^3+3^3+$ &c. to (n) terms, equals the square of the series $1+2+3+$ &c. to (n) terms.

9. Prove the following formulæ,

$$\sin. (x-z) = \text{ultimately } \sin. x - z \cos. x + \frac{z^2}{2} \sin. x,$$

(z) being diminished sine limite;

$$\frac{\cos. A + \sin. A}{\cos. A - \sin. A} = \text{tang. } 2A + \sec. 2A;$$

and having given an arc A, find another arc B so that tang. $B = \sec. A - \text{tang. } A$.

10. Force $\propto \frac{1}{(\text{dist.})^2}$; shew that the velocity at any point of the descent $\propto \tan. \left(\frac{\theta}{2}\right)$; θ being the circular arc whose diameter is the first distance, and versed sine the space described.

11. The increment of a semicircular area made by ordinates perpendicular to the diameter : the contemporary increment of the corresponding sector ::

versed sine of twice the arc : radius. Required proof, and find that point at which the difference between the sector and the area is a maximum.

12. Transform the equation $y^3+2py^2-33p^2y+14p^3=0$ into one whose coefficients are *numeral.*

13. Determine that point in an ellipse, force in focus, where the velocity is a harmonic mean between the greatest and least velocities.

14. Find the content of the greatest cone that can be cut out of a given paraboloid, the vertex of the cone being in the centre of the base of the paraboloid.

15. Find that point in P's orbit (11th section) at which the tangential ablatitious force : the mean addititious :: 3 : 2.

Monday Evening.—Mr. Jephson.

1. Extract the square root of $4mn+2.(m^2-n^2).\sqrt{(-1)}$.

2. There are (p) arithmetical progressions each beginning from unity, the common differences are 1, 2, 3.. . . .p, shew that the sum of their n^{th} terms $=\dfrac{(n-1).p^2+(n+1).p}{2}$.

3. Force to $S \propto \dfrac{1}{(\text{dist.})^2}$; prove that the velocity acquired in descending down any space AC : that which would have been acquired at C if the force

at A had continued constant :: the chord : the sine of a circular arc whose diameter is SA and versed sine AC.

4. If systems of logarithms be taken, whose bases increase in geometrical progression, shew that their moduli decrease in harmonical progression.

5. Given the sun's declination, and that the sun is due east when half the time between his rising and twelve o'clock is elapsed; find the latitude of the place.

6. There are two events A and B independent on each other; the probability of A's happening : probability of failing :: p : q; the probability of B's happening : probability of failing :: r : s. In $(2n)$ trials what is the probability that they will happen alternately all along?

7. Trace and construct the curve whose equation is $y^2 = \dfrac{x^3 + bx^2}{c - x}$ and determine the acute angle at which it cuts the axis.

8. The concave surface of a cylinder filled with fluid is divided by horizontal sections into (n) annuli in such a manner that the pressure upon each annulus is equal to the pressure upon the base; given the radius of the cylinder, find its height, and also the breadth of the (p^{th}) annulus.

9. Find $f.\ \dfrac{\dot{x}}{x.(a^2 + x^2)^{\frac{5}{2}}}$; also having given

$\int . (a+cz^n)^m . z^{vn-1}\dot{z}$, find $\int . (a+cz^n)^{m+p} . z^{vn-1}\dot{z}$; also find the values of y in the equation $y - \frac{\ddot{\ddot{y}}}{\dot{z}^4} = 0$.

10. Sum $\frac{5}{1.2.1.3} + \frac{9}{2.3.3.5} + \frac{13}{3.4.5.7} + \&c.$ to (n) terms and in inf. and

$$\frac{1}{1.3^2} + \frac{2}{3.5^2} + \frac{3}{5.7^2} + \&c. \text{ in inf.}$$

and apply the method of increments to sum (n) terms of the series

$$1 + \frac{3}{1.3} + \frac{3.4}{2.3^2} + \frac{4.5}{2.3^3} + \&c.$$

11. Find the equation to a spiral in which the angle described by the radius vector $SP \propto \frac{1}{SP^n}$; and shew that the subtangent to any point P : the corresponding circular arc described with radius SP and beginning from the asymptote :: n : 1.

12. A revolving spheroid will retain its form if four times the primitive gravity at the equator : five times the centrifugal force of rotation :: semi-axis : the elevation of the equator above the inscribed sphere.

13. Two equal weights sustain each other by means of three tacks situated in the same vertical plane; prove that the *vertical* pressures are together equal to the sum of the weights.

14. An inverted paraboloid is supplied with water at a given rate; given its dimensions, and the area

of the orifice which is in the vertex; it is required to find the highest point to which the water will rise, and also the time.

15. Parallel rays are incident upon a semi-cycloid in a direction perpendicular to the base; find the caustic, and shew that the density $\propto$ tang. $\theta +$ 2 tang. $2\,\theta$, (θ being that arc of the generating circle of the cycloid which corresponds to the point of incidence.)

16. If $(v) =$ the true anomaly, $u =$ the eccentric, and $e : 1$, the ratio between the eccentricity of the orbit, and its semi-axis major, then

$$\cos. v = \frac{e + \cos. u}{1 + e . \cos. u}.$$ Required proof.

17. A logarithmic curve being described, construct for its sub-tangent.

18. P and Q are placed at the ends of a lever, P hangs by a thread; given P and Q, it is required to find where the fulcrum must be placed so that the tension of the thread may be a maximum.

19. A body oscillating in a medium whose resistance $\propto$ (vel.)^2, construct for the resistance at each point. (Newt. Prop. xxix. Vol. II.)

20. By means of the compasses alone, it is required to inscribe in a square an equilateral triangle having one angle in an angle of the square.

21. The corner of a rectangular piece of paper is doubled down, so that the triangle shall always be of

a given area, prove that the vertex of the triangle will trace out a lemniscata, whose area equals the area of the triangle and which may be described by a force placed in its *knot* varying as $\frac{1}{D^7}$.

22. A quadrant is stretched out into a straight line, and upon it as an axis ordinates are drawn which are equal to the versed sine of twice the intercepted arc; find the whole area of the curve so traced out, and determine the point of contrary flexure.

23. The nodes being in quadratures, prove that the mean decrement of their motion arising from the acceleration of the areas is equal to $\frac{1}{4}$th of the decrement in syzygy.

24. If a perfectly flexible and uniform chain of a given weight coincide with the convex surface of a vertical quadrant having one radius horizontal, find the velocity acquired in its descent, and the tension at a given point in any given position of the chain.

Tuesday Morning.—Mr. Jephson.

FIRST AND SECOND CLASSES.

1. Are the quantities $\sqrt[4]{(-a)} \times \sqrt[4]{(-b)}$ and $\sqrt[6]{(-a)} \times \sqrt[6]{(-b)}$ possible or impossible?

2. Given that a parabolic area : its circumscribing parallelogram always :: $m : n$; it is required to find the ratio between the solids generated by these

areas revolving round their common axis, and to apply it to the case of the common parabola.

3. Shew that at a point of contrary flexure the fluxion of $\frac{\dot{x}}{\dot{y}} = 0$.

4. The inscribed sphere is taken away from an oblate spheroid of small eccentricity; find that annulus parallel to the plane of the equator which attracts a corpuscle at the pole with a maximum force.

5. If $(-a)$ be a root of the equation

$$x^n + px^{n-1} \mp qx^{n-2} \pm, \&c. \ldots\ldots \pm W = 0,$$

shew that $x+a$ is a divisor of the *expression*

$$x^n + px^{n-1} \mp qx^{n-2} \pm, \&c. \ldots\ldots \pm W.$$

6. The latitudes of two places on the same meridian are observed, and perpendiculars to their horizons are drawn meeting within the earth, (not supposed to be a sphere); shew that the angle at which the perpendiculars will meet is equal to the difference of the observed latitudes.

7. Find the content of the solid generated by the revolution of a cycloid round a tangent parallel to the base.

8. Explain the Nonius, and shew that the instrument is rendered more *sensible* by increasing the number of the divisions.

9. A bag contains three red balls and two white ones; what is the probability of taking out a white ball in two trials?

10. Give a geometrical construction for finding the resistance upon any curve, and apply it to determine the ratio between the resistance upon a catenary moving in the direction of its greatest ordinate and the resistance upon its axis.

11. The circumference of a semi-circle being considered as an abscissa, and ordinates drawn from its convex side in a direction perpendicular to the diameter and varying as the n^{th} power of the intercepted arc, it is required to draw a tangent to the quadratrix thus traced out, and to shew that in the case of the cycloid, the tangent is parallel to the corresponding chord of the generating circle.

12. A perfectly hard body falls down an inclined plane *AB*, and is inflected along another inclined plane *BC*; now *AD, DE, EF* are respectively perpendicular to *CB* produced, to *AB* and to the horizontal line *CF*; shew that the velocity acquired at *C* is equal to that which would be acquired by falling down *EF*.

13. Every recurring series whose scale of relation is $f-g+e$, may be resolved into three geometrical series, whose common ratios are the roots of the equation $x^3-fx^2+gx-e=0$.

14. *P* hanging freely raises *W* up an inclined plane by means of a thread not parallel to the plane; find the tension of the string.

15. If (t) = the time of a comet's passage between

its nodes, (q) = one year, $\pi = 3.14159$, then will the line of nodes $= \sqrt[3]{\frac{(18\pi^2 t^2)}{q^2}}$, the earth's mean distance being (1). Required proof.

Tuesday Afternoon.—Mr. Jephson.

FIFTH AND SIXTH CLASSES.

1. Shew that sin. $(-A) = -(\text{sin. } A)$, and that cos. $(-A) = \text{cos. } A$. Is sec. $(-A) = +$ or $-$ sec. A?

2. Solve the following equations:

(1.) $a + x + \sqrt{(2ax + x^2)} = b$

(2.) $$\left.\begin{aligned} \frac{y}{x} - \frac{x}{x+y} &= \frac{x^2 - y^2}{y} \\ \frac{x}{y} - \frac{x+y}{x} &= \frac{y}{x} \end{aligned}\right\}$$

3. The earth a sphere, and its radius 4000 miles, find what distance may be seen by a person 9 feet high.

4. Divide $x^3 - px^2 + qx - r$ by $x - a$.

5. If two equal forces sustain each other by means of a string passing over a tack, shew that either of the forces : pressure upon the tack :: radius : 2 cosine of half the angle at which the forces act.

6. If $a : b :: c : d$, then will $a \pm mb : b :: c \pm md : d$.

7. Prove that the radius of a circle which bisects the chord will bisect the arc.

8. Shew that the versed sines of the same angle in different circles are to each other as the radii of the circles.

9. If $A : B :: C : D$, then will $\log. D = \log. B + \log. C - \log. A$.

10. An inverted paraboloid is filled with a fluid; find that horizontal section which sustains the greatest pressure.

11. Explain the magic lantern.

12. Shew the use of logarithms in finding the value of $\frac{A^2 \sqrt{(B^2 - C^2)}}{C \sqrt[5]{(D^3 . E . F)}}$.

13. Find all the combinations which can be made out of the letters of the word *Baccalaureus*.

14. A certain velocity (a) is communicated to each of two perfectly hard bodies at the instant of their impinging on each other. Shew that the common velocity after impact equal $a \pm$ what would have been the common velocity if (a) had not been communicated.

15. Find the 28th term of the series 13, $12\frac{2}{3}$, $12\frac{1}{3}$, &c. and sum the series $2, -\frac{1}{3} + \frac{1}{18}, -\frac{1}{108}, +$ &c. in inf.

16. Take the fluxion of $(x^m + bx^n)^p$ and find the

$$f. \frac{\dot{x}}{a - mx}, \text{ and } f. \frac{z^{\frac{n}{2}-1} \dot{z}}{a^n + z^n}.$$

17. Prove that in all curves the centripetal force : the centrifugal :: $SP : \frac{1}{2}.\frac{QT^2}{QR}$.

18. Force to S varying as dist.; shew that the velocity acquired in descending through any space AC : that which would have been acquired at C if the force at A had continued constant :: sine : the chord of a circular arc whose radius is SA, and versed sine AC.

Tuesday Afternoon.—Mr. Macfarlan.

THIRD AND FOURTH CLASSES.

1. Solve the equation $x^3-11x^2+36x-36=0$, the roots being in harmonical progression.

2. If (n), a prime number, be the index of a binomial, every term of the expanded binomial, except the first and last, is divisible by (n).

3. If two bodies acted upon by constant moving forces in the proportion of 5 : 4, describe spaces from rest in the proportion of 4 : 5, and acquire velocities in the proportion of 5 : 6. Required the ratio of the quantities of matter.

4. In a given circle, the plane of which is vertical, to draw a diameter, which shall be described by a heavy body in any given time, not less than the time in which the vertical diameter is described.

5. The least angle which can be made with the horizon, by any great circle passing through the

place of a star at any given time, is measured by the star's altitude.

6. The periodic times in all ellipses round the same center are equal.

7. Find the effects of precession in right ascension and declination; and shew when the effect in right ascension vanishes.

8. Find the fluent of

$$\frac{x^3\dot{x}}{\sqrt{(2ax-x^2)}}, \text{ and of } \dot{z} \times (\sin. z)^2.$$

9. Sum the following series:

(1.) $1^2 + 3^2 + 5^2 + 7^2$ to 12 terms.

(2.) $1.x + 2x^2 + 3x^3 + 4x^4 \ldots + nx^n$.

(3.) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}$, &c. ad inf.

10. The density of the sun's rays formed by a spherical reflector : density of his rays formed by a glass sphere of the same radius :: 9 : 1. Required proof.

11. Compare, after Newton's method, the resistance upon the solid generated by the revolution of a cycloid round the base, moving in the direction of the base, with the resistance upon the circumscribing cylinder.

12. Having given the angle of a plane triangle, the side opposite to it and the sum of the other two sides; required the sides.

13. Let a sphere descend by its gravity in a fluid, whose specific gravity is to that of the sphere as 1 to n. Find the greatest velocity it can acquire on supposition that the resistance varies as the square of the velocity.

14. Extract the square root of 2 by a continued fraction.

Tuesday Evening.—Mr. MACFARLAN.

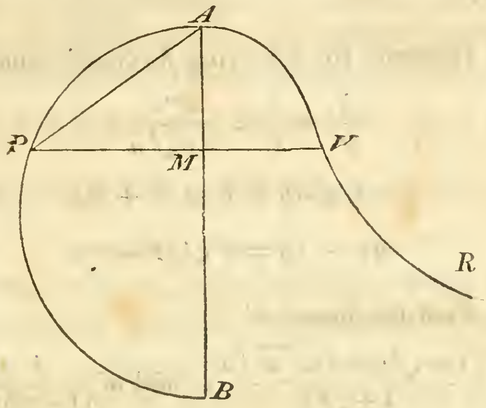
1. If the national debt be A£. and P£. be annually invested at compound interest as a sinking fund, in how many years will the debt be discharged, the interest of A not being considered?

2. Find the sun's place in the ecliptic, when the aberration of a given star in declination vanishes; and shew that the aberration in right ascension is not necessarily a maximum when the aberration in declination $=0$.

3. Find, after Newton's manner, the law of force whereby a body may describe a semi-ellipse, the direction of the force being perpendicular to the axis major.

4. When the force varies inversely as the fourth power of the distance, and a body is projected from an apse with the velocity acquired in falling from an infinite distance, to define the orbit.

5. The curve AVR and the semi-circle APB have the same abscissa; the ordinate MV is equal



to the tangent of half the arc AP. Prove that the area AMV is equal to twice the circular segment AP; and find the point of contrary flexure.

6. Sum the series

$$\frac{5}{1.2.3}+\frac{6}{2.3.4}+\frac{7}{3.4.5} \text{ to } n \text{ terms,}$$

and $$\frac{6^2}{1.2.3.4}+\frac{7^2}{2.3.4.5}+\frac{8^2}{3.4.5.6} \text{ to } n \text{ terms}$$

and ad infinitum.

7. Find the chance of throwing three aces exactly in five throws with a single die.

8. Find the greatest of all triangles having the same vertical angle and equal distances between that angle and the bisection of the opposite side.

9. In orbits of little eccentricity, the greatest equation of the center is equal to twice the eccentricity.

10. Integrate the following fluxional equations:

$$\frac{\dot{y}}{y} - \frac{\dot{x}}{x} = \frac{x^m \dot{x}}{ay\sqrt{n}};$$

$$x\dot{x} + a\dot{y} = b\sqrt{(\dot{x}^2 + \dot{y}^2)}.$$

$$y\dot{x} - x\dot{y} = \dot{x}\sqrt{(x^2+y^2)}.$$

11. Find the fluent of

$$\frac{\{1+\sqrt{(-x)}\sqrt[3]{x^2}\}\dot{x}}{1+\sqrt[3]{x}}, \text{ and of } \frac{\dot{x}}{(1+x^2)^n};$$

and when $n=3$ shew that the value of the latter fluent, between the value of $x=0$ and $x=1$, is $\frac{1}{4}+\frac{3A}{8}$, A being an arc of 45^0 to radius 1.

12. If any number of projectiles be thrown from the same elevated point, and with the same velocities in an horizontal direction; the locus of the points in which they will strike a given inclined plane will be a conical frustum.

13. If a cylindrical vessel, placed vertically, and kept full of water, be bored in innumerable points; the issuing fluid will be bounded by the surface of a conical frustum.

14. Find the equation between the abscissa and ordinate of the catenary.

15. A body (P) draws a lighter body (W) over a fixed pulley. A small oscillating motion is given to W at the commencement of P's action. Find the number of oscillations before W reaches the pulley: show that this number is independent of the string's length; and that however great P is, $\frac{1}{\sqrt{2}} \times$ oscillation at least will be performed in the time specified.

16. Two rods, the one 6, the other 8 feet high, are placed on a given day perpendicularly to the horizon, at the distance of 20 feet from each other. During the forenoon the extremity of the shadow of the first rod falls at the base of the second. In the afternoon the extremity of the shadow of the second falls at the base of the first. Required the latitude of the place, and the azimuth of one rod seen from the other.

17. If a body be projected from an apse with a given velocity, the force acting perpendicularly to a given plane, and varying in some inverse ratio of the distance from it, investigate the fluxional equation to the curve which will be described, and apply it to the case where the force is constant.

18. Supposing the sun to move uniformly on in a right line with a given velocity, and the earth revolving round him, to preserve always the same distance, it is required to define the earth's path in fixed space.

19. A body describes the quadrant of a circle touching a vertical line at its highest point, being urged by a force perpendicular to the horizon. Required the law of force which will make it recede uniformly from the horizontal radius, and the time elapsed and the velocity acquired at any point of the descent.

20. If any number, a multiple of 11, and a number consisting of the same digits in an inverted order, be each divided by 11, the sum of the digits in the two quotients are equal. Required a proof.

21. If n be a prime number, the product of $1 \times 2 \times 3 \times 4 \ldots \times (n-1)$ when increased by unity is divisible by n.

22. If the coefficients of each term of the expanded binomial $(a-b)^n$ taken in order, be multiplied by the terms of the progression 1^m, 2^m, 3^m, &c. taken in order, the result is equal to nothing, n and m being integers, and n greater than m. Required proof.

23. Let α, β, γ, &c. be the roots of the equation

$$x^n - nax^{n-2} + n.\frac{(n-3)}{2}a^2x^{n-4} - \text{\&c.} = -b\text{; then}$$

$$\alpha^{2r} + \beta^{2r} + \gamma^{2r} + \text{\&c.} = \frac{2r.(2r-1).(2r-2)\ldots(r+1)}{1.2.3\ldots\ldots\ldots\ldots r}$$

$\times na^r$ ($2r$ less than n.) Required a proof.

24. A body, whose weight is W falls down the length of an inclined plane, which has the power of

moving freely along an horizontal plane, on which it stands. Given the weight of the prismatic figure composing the plane, it is required to find the path of the body *W*, the time of describing it, and the last acquired velocity of the moveable plane along the horizontal plane.

1814.

Monday Morning —Mr. Bland.

Monday, January 17, 1814.

FIRST AND SECOND CLASSES.

1. The time of a body's falling through half radius by the uniform action of the centripetal force in the circumference of a circle is to the periodic time as radius is to the circumference of the circle. Required a proof.

2. Prove, without resolving the equation into factors, that if two numbers, a and b, when substituted for the unknown quantity in the equation $x^n - px^{n-1} + qx^{n-2} - \&c. = 0$, give results affected with contrary signs, there is at least one root between a and b.

3. If a line intercepted between the extremity of the base of an isosceles triangle, and the opposite side (produced if necessary) be equal to a side of the triangle, the angle formed by this line and the base produced is equal to three times either of the equal angles of the triangle.

4. Given the greatest range of a projectile upon an horizontal plane; determine at what distance from

the point of projection an object, whose perpendicular height is (d), must be situated, so that the projectile may just strike the top of it.

5. From the vertex of a paraboloid of given dimensions, a part equal to one-fourth of the whole is cut off by a plane parallel to the base; and the frustum being then placed in a fluid with its smaller end downwards, sinks till the surface of the fluid bisects the axis which is vertical. It is required to determine the specific gravity of the paraboloid, that of the fluid in which it is immersed and the density of the atmosphere being given.

6. Given the fluent of $\dfrac{d\dot{z}}{z\,.\,(a+c\,z^2)^2} = A$,

to find the fluent of $\dfrac{d\dot{z}}{z^3.(a+cz^2)^2}$;

and find the algebraical relation of x to y, in the equation $y^2\dot{y}=3yx\dot{x}-x^2\dot{y}$.

7. Find the sum of the series

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\text{\&c. in inf.}$$

and $\dfrac{10.18}{2.4.9.12}+\dfrac{12.21}{4.6.12.15}+\dfrac{14.24}{6.8.15.18}+$ &c. to n terms.

8. Require the nature of the curve such, that if any point P in it be taken, and an ordinate PN and normal PG be drawn to the axis; then if the triangle PNG be placed in such a position that the sub-normal NG may become the ordinate, PG will be the normal.

9. To spectators situated within the tropics, the sun's azimuth will admit of a maximum twice every day, from the time of his leaving the solstice till his declination becomes equal to the latitude of the place. Required a proof.

10. If parallel rays fall upon a single thin lens of given substance; determine the diameter of the least circle into which all the rays of different colours are collected, the linear aperture of the lens being known.

11. Compare the magnitude of that part of the disturbing force of the sun on the moon, which acts perpendicularly to the plane of the moon's orbit, with the moon's gravity to the earth.

12. The velocities of two bodies A and B are in a given ratio, and they begin to move at the same time from A and B, the extremities of a given line AB; A moving uniformly in a direction inclined at a given angle to AB, and B uniformly in the direction BA. Determine the nature of the curve, to which the line joining the bodies is always a tangent.

13. The moon's nodes complete a revolution in about 19 years. Determine the periodic time of the nodes of the third satellite of Jupiter, which revolves in about seven days, Jupiter's period being about 12 years.

14. If (a) be the number of chances for the happening of an event, and (b) the number for its

failure in each single trial; find the probability of its happening p times and failing q times in $(p+q)$ trials; and determine how many trials are necessary to make it an even chance whether the event will happen or not.

Monday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $\frac{9.0240160}{25.3009}$.

2. Prove the rule for completing the square in a quadratic equation.

3. Find the sum of the series

$$-9-7-4-\&c. \text{ to 20 terms,}$$

$$\text{and } 1+\frac{2}{3}+\frac{4}{9}+\&c. \text{ to 10 terms,}$$

$$\text{and } 1.2+2.3+3.4+\&c. \text{ to } n \text{ terms.}$$

4. If three quantities are in an increasing arithmetical progression; shew that the second will have to the first a greater ratio than the third to the second.

5. The weights of two perfectly elastic balls are 11 and 8, and their velocities in opposite directions are 12 and 7. Required their velocities after impact.

6. Let the height of an inclined plane be (a) feet, and its length (c) feet. Find the time of a body's descending down (a) feet of the plane.

7. Two fluids, whose magnitudes and specific gravities are given, being mixed together; the magnitude of the mixture : sum of the magnitudes of the ingredients :: n : 1. Determine the specific gravity of the mixture.

8. Compare the time in which any prismatic vessel is emptied by an orifice in the lower surface, with the time of a heavy body's falling through a space equal to twice the depth of the orifice.

9. Divide a right line into two parts, such, that their rectangle may be equal to a given square; and determine the greatest square that the rectangle can equal.

10. Find the fluxions of

$$(a+cz^n)^m \times zp, \text{ and of } x.\frac{\sqrt{(1+x^2)}}{\sqrt{(1-x^2)}};$$

and find the fluents of

$$\frac{\dot{x}}{x\sqrt{(x^2-a^2)}}, \quad \frac{x^3\dot{x}}{a^2-x^2}, \text{ and } x^2\dot{x}\sqrt{(a^2+x^2)}.$$

11. Construct Newton's Telescope, and find its magnifying power.

12. Explain the reason why the order of the colours is inverted in the secondary rainbow.

13. Given the sun's altitude at six o'clock, and also when due east; find the latitude of the place.

14. If from a quantity which varies as $\frac{1}{A^2}$, any

quantity be subtracted which varies as A, the remainder will vary in a higher inverse ratio than the inverse square of A; but if to a quantity varying as $\frac{1}{A^2}$ another be added which varies as A, the sum will vary in a lower inverse ratio than the inverse square of A. Required proof.

15. Find the law of the force tending to the centre of the logarithmic spiral.

16. Prove that when the force acts in parallel lines, the velocity in the direction perpendicular to the direction in which the force acts, is constant.

17. If the altitude of a cylinder be equal to the diameter of its base, the whole surface is six times the area of the base.

18. If $a^{mx} b^{nz}$ be constant, and $(mx+n).(nz+m)$ be a maximum; prove that $a^{mx+n} = b^{nz+m}$.

Monday Afternoon.—Mr. MACFARLAN.

THIRD AND FOURTH CLASSES.

1. Required the perpendicular from the vertex upon the base of a triangular pyramid, all the sides of which are equilateral triangles of a given area.

2. Given the difference of the times of setting of two stars whose declinations are known; find the latitude of the place.

3. Find the center of oscillation of a conical surface suspended by the vertex; and find the ratio

between the radius of the base and the axis, when the center of oscillation is in the base.

4. The length of a pendulum oscillating seconds on the earth's surface being given; find the length of a pendulum oscillating seconds at the distance of the earth's radius from the surface. Then determine a point below the surface where the last pendulum will vibrate in the same time.

5. Two roots of the equation $x^4+x^3-11x^2+9x+18=0$ are of the form $+a$, $-a$. Find all the roots.

6. When the force varies as that power of the distance whose index is $(n-1)$. Shew that the velocity of a body falling from rest varies as $\sqrt{\frac{(P^n-A^n)}{n}}$, where P is the greatest and A the variable distance. And find the value of this expression, when the force varies inversely as the distance.

7. If from the extremity of the diameter of a circle tangents be drawn and produced to intersect a tangent to any point in the circumference, the right lines joining the points of intersection and the center of the circle shall form a right angle.

8. Sum the series

$$\frac{1}{1.3}-\frac{1}{2.4}+\frac{1}{3.5}-\frac{1}{4.6}+\text{\&c. to } n \text{ terms,}$$

and $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\frac{1}{8.9}$ ad inf.

9. Find the fluents of

$$\frac{\dot{x}}{\sin. z \times \cos. z}, \text{ and } \frac{\dot{x}}{\sqrt{(A+Bx-Cx^2)}}.$$

10. Find the attraction of a sphere on a particle of matter placed at any distance from the center, the force of each particle varying inversely as the cube of the distance.

11. Find the equation to the curve, the length of whose tangent between any point and the axis is a constant quantity.

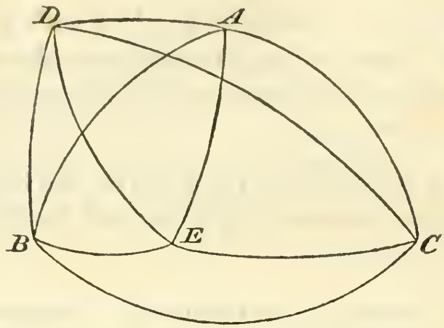
12. The equation to a curve is $y^3 - axy + x^3 = 0$. Find the value of the ordinate when a maximum, and the corresponding value of the abscissa; and show that the above is a maximum and not a minimum.

13. A paraboloid placed with its vertex downwards being full of water, is supplied at a given rate. There is a small hole in the vertex, which, when the vessel is full, would discharge n times the quantity supplied. Required the altitude at which the surface remains stationary, and the time elapsed before this takes place.

Monday Evening.—Mr. Macfarlan.

1. A body placed in the center of gravity of a triangle is acted upon by three forces represented in quantity and direction by the lines joining the center of gravity with the three angles. Show that the body will remain at rest.

2. The sides of a spherical triangle ABC are each a quadrant. D and E (any two points on the surface of the sphere) are joined by the arc of a



great circle. Show that the cosine of DE is equal to the cos. $AD \times$ cos. AE + cos. $BD \times$ cos. BE + cos. $CD \times$ cos. CE.

3. If the sum of the odd digits in a number be $11m+e$ and of the even $11n+e$, this number being divided successively by 11 and by 9, leaves the same remainder as $m+n+e$ when divided by 9.

4. In a dial for a given latitude, the plate of which ought to have been horizontal, the interval between ten and noon is less by two minutes, than the interval between noon and two o'clock. The line between north and south was found to be horizontal. Required the dip of the plate towards the east.

5. A sphere filled with water empties itself through a small hole in the bottom; find where the velocity of the surface of the descending fluid is a

minimum, and where it is equal to the velocity when the sphere is half full.

6. The mean apparent diameter of the sun and moon's horizontal parallax being given, together with the length of a year and a month, find the density of the sun compared with the density of the earth; also shew how Newton finds the density of the moon.

7. From Newton's construction for the solid of least resistance, shew that in the section through the axis, the curve must make with the end an angle of 135^0.

8. If the quiescent orbit be a circle (the center of force in the circumference) and the angular velocity in the moveable orbit is double that in the quiescent; Find the law of force in the orbit in fixed space, and investigate the ratio between the perpendicular and distance.

9. A wooden ball connected by a small wire with a ball of lead of the same diameter is dropt into the sea, and upon their striking the bottom, the wooden ball is disengaged and rises to the surface; the whole time elapsed, and the specific gravities and diameters of the balls being given; find the depth of the sea.

10. In any conic section, if tangents be drawn at the extremities of any diameter, and be produced to meet a tangent to any other point in the curve; prove that the rectangle under the segments of the

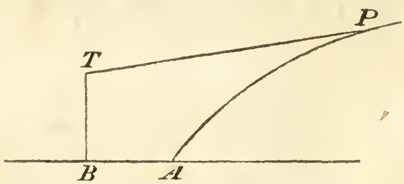
first tangents will be equal to the square of the semi-conjugate diameter.

11. $n^n - n(n-2)^n + n \cdot \frac{n-1}{2} \cdot (n-4)^n - n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} \cdot (n-6)^n$ &c. continued to $\frac{n}{2}$, or $\frac{n+1}{2}$ terms, is equal to $1 \times 2 \times 3 \times \ldots\ldots n \times 2^{n-1}$. The demonstration is required.

12. A given number (n) of similar balls being put into an urn; required the chance of drawing an odd, to the chance of drawing an even number; any number, from 1 to n inclusive, being equally likely to be drawn.

13. Find the equation and construct the curve of which this is the property: if from a fixed point in the axis a perpendicular be drawn to it, and produced to meet a tangent to any point in the curve, the length of this perpendicular and tangent together shall be double the length of the curve between the vertex and the point from which the tangent was drawn.

In the figure annexed, $BT+TP=2AP$.



Does the curve admit of an asymptote?

14. In a medium resisting as the square of the velocity, show that a perfectly elastic body falling from an infinite height will at each rebound rise through spaces proportional to the logarithms of the fractions $\frac{2}{1}, \frac{3}{2}, \frac{4}{3} \ldots\ldots\ldots\ldots \frac{n+1}{n}$.

15. Sum the series

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \text{\&c. ad inf.}$$

and $1^3 + 3^3 + 5^3 + 7^3$ to n terms, by increments.

16. Given the fluent of $(e + fx^n)^m \times x^p \dot{x}$, find the fluents of $(e + fx^n)^m \times x^{p+n}\dot{x}$,

and of $(e + fx^n)^{m+1} \times x^p \dot{x}$;

also find the fluent of $\dfrac{a^{\frac{1}{2}} + y^{\frac{1}{2}}}{y^{\frac{1}{2}} + y^{\frac{2}{3}}} \times \dot{y}$.

17. Solve the following fluxional equations,

$$x^2\dot{y} + 2xy\dot{y} = b^2\dot{x} - y^2\dot{y}$$
$$cx^2\dot{x} + y\dot{x} = a\dot{y}.$$

18. Show that the log. $(1 + n . \cos. z)$ is equal to $A + \frac{2B}{1}\cos. z + \frac{2B^2}{2}\cos. 2z + \frac{2B^3}{3}\cos. 3z + \frac{2B^4}{4} \times \cos. 4z$ &c. where A is the log. of $2B$, and $B = \frac{1 - \sqrt{(1 - n^2)}}{n}$.

19. In a medium resisting as the square of the velocity, find the nature of the curve to be described by a heavy body urged by the force of gravity, so that the times of descent through different arcs

to the same fixed point shall vary as the velocity acquired.

20. Four persons (*A*, *B*, *C*, *D*) to whom the cards of a common pack are dealt in order, one by one, stake each £1. with the condition, that he to whom the first knave is dealt, shall be the winner. What is the value of *A*'s expectation?

21. Find the curve by the revolution of which round an axis the solid will be formed, which shall attract a particle placed at its vertex with the greatest possible force, the force of each particle varying inversely as the square of the distance.

22. A cylindrical vessel full of water is balanced by a weight *P* over a fixed pulley. A hole of given dimensions being made in the bottom, it is required to find how far *P* will descend during the time of emptying.

23. Prove that the sum of the reciprocals of the prime numbers is an infinitely great number, though infinitely less than the sum of the reciprocals of the natural numbers.

Tuesday Morning.—Mr. Macfarlan.

FIRST AND SECOND CLASSES.

1. A person borrowed *P* £. at interest. To discharge this he invested £2. at the end of the first year, £4. at the end of the second, and 8£. at the

end of the third, and so on. How many years will elapse before this fund be large enough to discharge the debt,—compound interest being allowed on both sides at a given rate?

2. Required the length of a spherical shell of iron, which, when filled with a fluid, shall just float in water; the specific gravities of iron, of water, and of the fluid, being given.

3. Compare the length of a degree of latitude at any place on the earth's surface, with the length of a degree of longitude at the equator.

4. The inclination of a small tube in the side of a vessel of water being given, and its height above the horizontal plane; it is required, from observing the point of the plane struck by the stream, to assign the altitude of the water within the vessel; and to describe the whole track of the issuing fluid.

5. If round any point within the circumference of a circle (not being the center) equal adjoining angles be described; of the circumferences on which they stand, that which is nearer the diameter passing through the point is less than the more remote.

6. In a combination of two wheels and axles, the circumference of each wheel is n inches; of each axle, 1. A weight, P, is applied to the circumference of one of the wheels as a power to raise matter to a certain height. How much must be raised each time, that the whole quantity may be raised in the least time possible?

7. Of all cones containing a given quantity of matter, to find that which attracts a particle placed at its vertex with the greatest possible force.

8. Show that when a quantity is a maximum or a minimum, the first fluxion vanishes; and that the quantity is a maximum or a minimum, accordingly as the second fluxion is negative or positive.

9. An imperfectly elastic ball is projected with a given velocity against an hard horizontal plane, and being reflected, just reaches the point of projection in t''. Required the distance of the plane, from the point of projection, and the elasticity of the body.

10. A cylindrical tube of given length, closed at one end, being let down in a vertical position into the sea, it was observed what part of the tube the water occupied. It is hence required to assign the depth, 33 feet of sea water being assumed to measure the weight of the atmosphere. How must this tube be graduated to be used as a gage to measure depths in the sea?

11. Find the length of a common parabola, and deduce Cotes's construction.

12. If $x = \frac{\epsilon^{az} - \epsilon^{bz}}{2a}$, where a and b are roots of the equation, $v^2 + 1 = 0$:

$$\text{Show that } \dot{z} = \frac{\dot{x}}{\sqrt{(1 - x^2)}}.$$

13. Sum the series

$$1^2+3^2+7^2+15^2 \text{ \&c. to } n \text{ terms,}$$

and $\frac{1}{2.3}\times\frac{1}{2}+\frac{1}{3.4}\times\frac{1}{2^2}+\frac{1}{4.5}\times\frac{1}{2^3}+$ &c. ad inf.

Tuesday Afternoon.—Mr. Macfarlan.

FIFTH AND SIXTH CLASSES.

1. Find the value of £. 1234141414l &c.

2. The amount of £500. in $\frac{3}{4}$ of a year was £520. Required the rate per cent.

3. Find the circumference of a circle whose radius is unity.

4. Sum the following series,

(1.) $2+3+\frac{9}{2}+$ &c. to 20 terms,

(2.) $\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+$ &c. to n terms,

and $\frac{1}{1.2.3}+\frac{1}{3.4.5}+$ &c. ad inf. by increments.

5. Solve the following equation whose roots are in arithmetic progression;

$$x^3-9x^2+23x-16=0.$$

6. Find the fluents of $\frac{a\dot{x}}{a^2-x^2}$, $\frac{x^2\dot{x}}{\sqrt{(a^2+x^2)}}$,

and the fluxion of $\frac{(x+a)^2}{\sqrt{(x^2-a^2)}}$.

7. The arc of a circle which a body, acted upon by a centripetal force, uniformly describes in any given time is a mean proportional between the diameter of the circle, and the space described by a heavy body from rest in the same time when urged by the force in the circumference continued uniform.

8. Show that the logarithm of $(1+u)$ is equal to

$$u - \frac{u^2}{2} + \frac{u^3}{3} - \frac{u^4}{4} + \frac{u^5}{5} - \text{\&c.}$$

9. Given the radii of the surfaces of a double convex lens and the ratio of the sines of incidence and refraction. Find the focal length.

10. Find the latitude of the place at which the sun sets at three o'clock on the shortest day.

11. When the force varies as the distance, the periodic time in all ellipses round the same center are equal.

12. A body (A) weighs 12lbs. in vacuo, and 9lbs. in water; another body (B) weighs 10lbs. in vacuo, and 8lbs. in water. Compare their specific gravities.

13. If the number of mean proportionals interposed between two elastic bodies A and X be increased without limit, the velocity of A will be to the velocity communicated to X by means of the intermediate bodies :: $\sqrt{X} : \sqrt{A}$.

14. The apparent diameter and declination of the sun being given, find the time of his transit over the meridian.

15. The plane of a circle being vertical, and any number of chords being drawn to the lower extremity of the vertical diameter; find the locus of any number of heavy bodies falling together from the upper extremities of the diameter and the chords at any given instant of time.

16. If any number of projectiles be thrown at the same instant from the same point and with equal velocities, but in several directions in the same vertical plane, they will at the expiration of any time all be found in the circumference of some circle.

Tuesday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Shew from the principles of the fifth book of Euclid, that a ratio of greater inequality is diminished, and of less inequality increased, by adding a quantity to both its terms.

2. The time of day at a given place determined from observations of the sun's altitude is $9^h. 10'. 45''$; and a chronometer set to Greenwich time shews $6^h. 3'. 10''$. Required the longitude of the place of observation from Greenwich.

3. In any harmonic progression, the product of the two first terms is to the product of any two adjacent terms as the difference between the two first is to the difference between the two others. Required a proof.

4. An object is placed between two plane reflectors, which are inclined to each other at an angle of 60^0. Determine the whole number of images formed by the reflectors.

5. If the greatest possible rectangle be inscribed in the quadrant of a given ellipse, shew that the elliptic areas cut off by the sides of the rectangle are equal.

6. Prove that an equation of an odd number of dimensions, and an equation of an even number if its first and last terms be of different signs, must have at least one real root.

7. Find the sum of the series

$$1.2.4 + 2.3.5 + 3.4.6 + \&c. \text{ to } n \text{ terms.}$$

$$\frac{1}{3} + \frac{1}{2} + \frac{3}{4} + \frac{9}{8} + \&c. \text{ to } n \text{ terms.}$$

and $1.2 + 2.3x + 3.4.x^2 + \&c.$ in inf.

8. If the force vary inversely as the square of the distance, and a body be projected at a given angle with a velocity which is to the velocity in a circle at the same distance, as $\sqrt{2} : 1$. Determine the nature of the orbit described.

9. Prove that the surface of any segment of a

sphere cut off by two parallel planes is to the whole surface of the sphere as the intercepted portion of the diameter is to the whole diameter.

10. Find the fluent of $\frac{(x-1).\dot{x}}{(x^2+1)^2}$; construct the fluent of $\frac{x\dot{x}}{x^2-2ax+1}$, where a is less than unity; and shew that the fluent of $\frac{d\dot{z}}{\sqrt{(a^2-bz^2)}} = \frac{d}{\sqrt{b}} \times$ circular arc whose sine is $\frac{z\sqrt{b}}{a}$ and radius $= 1$.

11. A paraboloid whose vertex is downwards is filled with water to a given altitude. Having given the diameter of the upper surface, find what ought to be the diameter of the hole at the bottom, so that the upper surface may descend through a given space in a given time.

12. If the force varies as the distance, and two bodies fall towards two different centres of force; compare their velocities at any points of their descent.

13. Two elastic balls, beginning to descend from different points in the same vertical line, impinge on a perfectly hard plane inclined at an angle of 45°, and move along a horizontal plane with the velocities acquired. Shew that if a circle be described, passing through the two points from which the balls began their motion, and touching the horizontal plane, the point of contact will bisect the dis-

tance between the vertical line and the point where they impinge on each other.

14. Given, that the distance of the centre of gravity of an area from its vertex is an (n^{th}) part of the abscissa; find the distance of the centre of gravity of the solid formed by the revolution of this area round its axis.

15. Determine the proportion between the radius of the globe and wheel, when the length of the cycloid within the globe (Sect. 10.) is a maximum.

16. If centripetal forces tend to the several points of spheres, proportional to the distances of those points from the attracted bodies, the compounded force with which two spheres will attract each other mutually is as the distance between the centres of the spheres.

Tuesday Evening.—Mr. Bland.

1. The sum of n arithmetic means between 1 and 19 is to the sum of the first $n-2$ of them :: 5 : 3. Find the means.

2. Two equal weights are connected by a string passing over a fixed pulley. Supposing a weight to be added on one side, and the length and weight of the string, and the difference of the altitudes of the weights at the commencement of the motion to be given; determine in what part of the descent, the

velocity will be neither increased nor diminished by the string's weight.

3. If the abscissa of a curve bear a finite ratio to the ordinate, prove that the abscissa will cut the curve in a finite angle.

4. The place of the node and the inclination of the moon's orbit to the plane of the ecliptic being given; find the place of the moon when her declination is the greatest possible.

5. Find the value of

$$\frac{\sqrt{(2a^3x-x^4)}-\sqrt{(ax^3)}}{a-\sqrt{(ax)}}, \text{ when } x=a;$$

and find the fluxion of

$$\text{the hyp. log. } \frac{\sqrt{(a+x)}+\sqrt{(a-x)}}{\sqrt{(a+x)}-\sqrt{(a-x)}}.$$

6. If, in a circle a straight line be drawn cutting the diameter at any angle (A); prove that the difference of the segments of the diameter will be to the difference of the segments of the line as the diameter is to the chord of an arc, which measures twice the complement of A.

7. If, from the extremity of the major axis of an ellipse which is perpendicular to the horizon, chords be drawn making with it angles of 75^0 and 45^0, and from the points where the chords meet the curve, ordinates be drawn to the axis; the square of the time down the first chord will be twice the square of the time down the second in the sub-

duplicate ratio of the rectangles under the segments of the axis made by the ordinates.

8. If a square be inscribed in a circle and another circumscribed about both: compare the pressures upon the circle and the squares when immersed vertically in a fluid; the angular point of the circumscribing square coinciding with the surface of the fluid.

9. A hollow cone, whose vertical angle is 60°, is filled with water, and placed with its base downwards. It is required to determine the place where a small orifice must be made in its side, so that the issuing fluid may strike the horizontal plane in a point, the distance of which from the bottom of the vessel is to the distance of the orifice from the top as 5 : 4.

10. The distance of the center of gravity of the surface of a solid from the vertex is equal to half the abscissa; determine the nature of the curve by the revolution of which round its axis the surface was generated.

11. If two equal parabolas be placed in such a manner that they may touch each other at the vertices, and one be made to roll upon the other, its focus will describe a right line; and the vertex cissoid, the diameter of whose generating circle is equal to half the latus rectum of the parabola.

12. If a body revolve in an orbit round a center

of force, and at the same time the orbit revolve round the same center in such a manner that the angular velocity of the body in the orbit if fixed, may be to its angular velocity when revolving, in the ratio of $F : G$; find the centripetal force necessary to retain the body in a revolving orbit, the force in the fixed orbit varying as the n^{th} power of the distance. And apply this to the cases of an ellipse when the center of force is in the focus, and when in the center.

13. Let α, β, γ, &c. be the roots of the equation $x^n - px^{n-1} + qx^{n-2} -$ &c. $= 0$, (m) of which are possible; shew that if the equation be transformed into one whose roots are $(\alpha-\beta)^2$, $(\alpha-\gamma)^2$, $(\beta-\gamma)^2$, &c. the last term of the transformed equation will be positive or negative according as $m.\frac{m-1}{2}$ is an even or an odd number.

14. Find the fluents of $\frac{e^z z \dot{z}}{(1+z)^2}$, where $e =$ the base of hyp. logs;

$$\frac{dy^2\dot{y}}{(ar^2+by^2).\sqrt{(r^2-y^2)}}, \quad \frac{\dot{x}}{\sin.^2 x \times \cos.^4 x},$$

and $$\frac{b\dot{y}}{(a^2-y^2).(a+y)^{\frac{2}{3}}}.$$

15. Find the relation of x to y in the equations,

$$y^n\dot{y} - a^{n-1}x\dot{y} + c^{n-1}y\dot{x} = 0,$$
$$a^2\dot{y}^2 + bx\dot{y}^2 = c^2\ddot{y},$$

and determine the nature of the curve, in which $\frac{\dot{x}}{x} - \frac{\dot{y}}{y} : \frac{\dot{x}}{y} - \frac{\dot{y}}{x} :: n : 1$, x and y being the abscissa and ordinate.

16. Find the sum of the series,

$$\frac{5}{1.2.3} + \frac{7}{2.3.4} + \frac{9}{3.4.5} + \frac{11}{4.5.6} + \&c.$$

to n terms by increments;

$$\frac{2}{3.5} - \frac{3}{5.7} + \frac{4}{7.9} - \frac{5}{9.11} + \&c. \text{ in inf.}$$

$$\frac{\text{tang. } A}{1} - \frac{\text{tang. } 2A}{2} + \frac{\text{tang. } 3A}{3} - \&c. \text{ in inf.}$$

and having given the sum of the series

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \&c. \text{ in inf.},$$

find the sum of

$$\frac{1}{1^2.2.3} + \frac{1}{2^2.3.4} + \frac{1}{3^3.4.5} + \&c. \text{ in inf.}$$

17. The reflecting curve is a semicircle, and the radiating point is in the circumference; determine the nature of the caustic, its length, and the density at different points.

18. The latitudes of two places on the earth's surface are complements to each other, and on a given day the sun rises (n) hours sooner at one place than at the other; determine the latitude of each place.

19. If a system of bodies be connected together and supported at any point which is not the center

of gravity, and then left to descend by that part of their weight which is not supported; $4\,l$ multiplied into the sum of all the products of each body into the space it has perpendicularly descended will be equal to the sum of all the products of each body into the square of its velocity; l being $=16\frac{1}{12}$ feet.

20. A ring of given weight descends by its gravity down the arc of *any algebraic curve;* and the curve revolves uniformly about its axis which is perpendicular to the horizon in t''. Determine the velocity of the ring at any point of its descent.

21. If the force of gravity be uniform, and act perpendicularly to the horizon, determine the path of a projectile in a medium, the resistance of which is proportional to the velocity of the body.

22. Having given the relation between the centrifugal force and the force of gravity at the earth's equator; determine the relation between the centrifugal force and the force of gravity at the equator of Jupiter; the densities and times of revolution round their axes being known.

23. Shew that the mean motion of the nodes of the lunar orbit is not affected by the excentricity of the orbit; and that the true motion of the nodes in an elliptic orbit is equal to the motion of the nodes in a circular orbit when the radius vector is a mean proportional between the semi-axes major and minor.

24. If an hyperboloid and a cone be generated by the revolution of an hyperbola and its asymptotes; and the cone being excavated and placed with its axis vertical, water be poured into it till the surface touches the vertex of the hyperboloid; shew that whatever be the inclination of the axis, the surface of the water will always be a tangent plane to the hyperboloid.

1815.

Morning Problems.—Mr. Hustler.

Monday, January 16, 1815.

FIRST AND SECOND CLASSES.

1. If A be any arc whatever, prove that cosec. A + cosec. $2A$ + cosec. $4A$ + &c. to n terms $= \text{cot.}\ \frac{A}{2} - \text{cot.}\ 2^{n-1}A$.

2. Shew that the sun's rising is least accelerated by refraction at the time of the equinoxes.

3. If an hyperbola and its asymptotes revolve about the axis major, prove that all the sections of the cone made by planes touching the hyperboloid have the same axis minor, which is the axis minor of the hyperbola.

4. Near the solstice the variations of the sun's declination are as the squares of the variations in longitude nearly.

5. Find from Taylor's Theorem the arc in terms of its *cosine*.

6. Having given the refracting powers of two mediums, find the ratio of the focal lengths of a convex and concave lens formed of these substances, which, when united, produce images nearly free from colour.

7. If t = tangent of half the angle ASP (Newton, Sect. 6.) to $r=1$, shew that the parabolic area $ASP=a^2\left\{\frac{t^3}{3}+t\right\}$, where a is the focal distance of the vertex.

8. If a caustic be formed by a reflecting curve, shew that the reflected ray is always a tangent to the caustic.

9. In any recurring series $a+bx+cx^2+$ &c. whose scale of relation is $f+g+h+$&c., if any row of the differences of a, b, c &c. = 0, prove that $f+g+h+$&c.$=1$.

10. If a cylinder be cut by two parallel planes intercepting a given part of the axis, shew that the solidity between the planes is the same whatever be the inclination of the planes to the axis.

11. Find the fluent of

$$\frac{\dot{x}}{x^3(a^2-x^2)}, \text{ and of } x^3\dot{x}\sqrt[3]{(a^2+x^2)}.$$

12. Having given the latitude of the place and the moon's declination, determine the height of the superior and inferior tide; and compare the height

of the tide at the equator and the pole of the earth, when the moon's declination is 30°.

13. If the moon's orbit be considered circular, and the position of the nodes be given, shew that, when the *horary* motion of the nodes is a maximum, the moon's distance from quadratures equals half the node's distance.

14. A semicubal parabola moves in its own plane, with its axis always coinciding with the same line. Determine the nature of a curve which, beginning at the vertex of the parabola in its first position, is perpendicular to it in all positions.

Monday Afternoon.—Mr. Hustler.

FIFTH AND SIXTH CLASSES.

1. If four quantities be in geometric progression, the sum of the two extremes is greater than the sum of the two means.

2. From the equation $a^{mx} = b - a^{mx-1}$, find the value of x.

3. If the interior angle BAC and the exterior angle DAC of any triangle ABC be bisected by lines AE, AF, which also cut BC in E, F, shew that BF, BC, BE are in harmonic progression.

4. Required the number of guineas with which

four persons A, B, C, D respectively begin to play, if after A has won half of B's, B a third of C's, and C a fourth of D's, each has twelve guineas.

5. What power acting parallel to the length of an inclined plane whose elevation is 30°, will draw a given weight Q, 40 feet up the inclined plane in 5″?

6. The latus rectum of an ellipse, being produced both ways to meet the circle described on the axis major, = axis minor.

7. Given the cot. A and cot. B, find cot. $(A \pm B)$ radius being $= 1$; and adapt the expression to radius r.

8. Form the biquadratic equation, two of whose roots are $1 + \sqrt{(a^3)}$ and $-\sqrt{(-b)}$.

9. How far must a body fall internally towards the focus of an hyperbola to acquire the velocity in the curve?

10. The sun's altitude at any time being 30°, find the position of a stick of given length so that the shadow may be the longest possible; and find the length of the shadow at that time.

11. Find the solid traced out by a curve whose equation is $y^2 = \dfrac{b^2x}{a-x}$.

12. If a billiard-table be in the form of an ellipse, and a perfectly elastic ball be struck from either

focus in any direction, it will return after two reflections from the curve to the same point.

13. An object at the bottom of a vessel appears to change its place when water is poured into the vessel. Explain this circumstance.

14. Given the specific gravities of wood and water :: 2 : 3, to what depth would a given paraboloid of wood sink when immersed with its vertex downwards?

15. Find the fluxions of

$$\frac{a+x}{a^2+x^2}, \text{ of } x.(a^2+x^2).\sqrt{(a^2-x^2)},$$

and of the secant of x; also the fluents of

$$\frac{\dot{x}\sqrt{(a^2-x^2)}}{x}, \text{ and of } \frac{b\dot{x}}{\sqrt{(1-ax^2)}}.$$

16. At a given place, and on a given day, find the point of the horizon where the sun rises.

17. Shew, according to Newton's second section, that if a parabola be described by a force tending parallel to the axis to a point indefinitely distant, the force must be constant.

Monday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Extract the square root of $ab-d^2+4c^2 \pm 2\sqrt{(4abc^2-abd^2)}$.

2. The sum of an even number of terms of any arithmetic progression, whose common difference is equal to the least term, will be four times the sum of half that number of terms diminished by half the last term.

3. The greatest and least corrected zenith distances of a circumpolar star being $38^0.\ 19'.\ 43''$, and $34^0.\ 53'.\ 49''$; determine the latitude of the place of observation.

4. Two forces acting upon a body in the same or in opposite directions, will cause it to move with a velocity equal to the sum or difference of the velocities which it would have received from the forces separately. Required a proof.

5. If bodies fall towards different centres of force, from different altitudes, compare the times of descending through any space; $F \propto \frac{1}{(\text{dist.})^2}$.

6. The length of the subtangent to the cissoid being equal to one fourth of the diameter of the generating circle; determine the point in the curve from which the tangent is drawn.

7. Transform the equation $x^3 - px^2 + qx - r = 0$ into one whose roots shall be mean proportionals between the roots of the equation, and a given quantity (m).

8. One side of a cubical vessel of water of given dimensions being loose; find the position, magni-

tude, and direction, of a single force which shall keep it at rest.

9. Find the time in which a vessel formed by the revolution of a given logarithmic curve round its asymptote, will empty itself through a given orifice in the bottom; the length of the axis and extreme ordinate being known.

10. If the force of gravity $\propto \frac{1}{(\text{dist.})^3}$, from the centre; find, according to Newton's method, the absolute velocity in feet, and absolute time in seconds, of descending through any space towards the centre.

11. Find the fluents of

$$z\dot{z}\sqrt{\frac{a+z}{a-z}}, \quad \text{and} \quad \frac{\dot{z}}{z}.(a^2+z^2)^{\frac{3}{2}};$$

and shew that the fluent of

$$\frac{\dot{z}}{(1-az^2).\sqrt{(1-z^2)}} = \frac{1}{\sqrt{(1-a)}} \times \text{circular arc},$$

whose radius $= 1$, and cosine $=\sqrt{\frac{1-z^2}{1-az^2}}$.

12. The focal length of a double convex lens, whose thickness is inconsiderable, and whose surfaces have the same curvature, is equal to the diameter of one of the surfaces. Determine the ratio of the sines of incidence and refraction.

13. The areas of unequal ellipses are in a ratio compounded of the subduplicate ratio of their para-

meters and the sesquiplicate ratio of their principal axes. Required a proof.

14. If a body be acted upon by two forces which vary according to different laws of its distance from the centre, as the p^{th} and q^{th} powers; determine the angle described, while it passes from one apse to the other; the orbit described being nearly circular, and the forces at the apse being as $1 : n$.

15. Let a plane isosceles triangle vibrate edgeways, suspended by its vertex. At what distance from its vertex must it strike an immoveable obstacle, so that its motion in the plane of vibration may be destroyed?

16. If two similar mediums are separated from each other by a space terminated on each side by parallel planes; and a body in its transit through this space, is attracted or impelled perpendicularly towards either medium, and is not agitated or hindered by any other force; and the attraction is every where the same at equal distances from either plane, taken towards the same side of the plane; prove that the velocity of the body before incidence is to its velocity after emergence as the sine of emergence to the sine of incidence.

Monday Evening.—Mr. Bland.

1. If (a) and (b) be the two first terms of a decreasing geometric progression, the sum of the series in inf. is $= \frac{a^2}{a-b}$. Required a proof.

2. If a tangent be drawn to any point of an ellipse, and from the point of contact a straight line be drawn to either focus; this shall be parallel to the straight line drawn from the centre to the intersection of the tangent and perpendicular from the other focus.

3. The moon's longitude at noon at Greenwich, January 1815, is

on the 16th. . . . $0^0, 16', 48''$.
17th. . . . $12^0, 46', 55''$.
18th. . . . $25^0, 35', 31''$.
19th. . . . $38^0, 46', 4''$.

Find its longitude on the 17th, at six o'clock.

4. In a given latitude, determine the vertical circle in which the difference of the altitudes of the sun in any two given days shall be a maximum.

5. If a body revolve in a reciprocal spiral, the force tending to the centre; prove that the times of its moving through successive angles of 180^0, are in the proportion of the numbers $\frac{1}{1.2}$, $\frac{1}{2.3}$, $\frac{1}{3.4}$, &c.

6. Prove that every odd cube number is equal to the sum of as many terms of a series, which have a common difference unity, as its root contains units, the middle term of the series being the square of the root.

7. Find the value of

$$\frac{1}{x+a}+\frac{a}{(x+a).(x+b)}+\frac{ab}{(x+a).(x+b).(x+c)}+\&c.$$

to n terms; and of

$$\frac{4}{1.3}-\frac{12}{5.7}+\frac{20}{9.11}-\frac{28}{13.15}+\&c. \text{ in inf.}$$

8. On the side of a vessel filled with fluid, let any number of circles be so situated that the pressures on them may be as the cubes of their diameters; determine the ratio of their distances from the surface of the fluid.

9. If water ascend and descend in the erect legs of a cylindrical canal, alternately; determine the nature and dimensions of the curve described by the centre of gravity of the water in the legs.

10. Two chains of the same uniform thickness and density are suspended from two given points, and attracted towards a centre of force, the law of the force being any power or root of the distance. Shew that the pressures on the points of suspension are proportional to the squares of the velocities which would be acquired by bodies falling towards the centre from the points of suspension, down spaces which are equal to the length of the chains.

11. Trace the curve whose equation is $x^2+y^2=\frac{b^2x^2}{2ax-x^2}$, and find its area when $b=a$.

12. A perfectly elastic ball A falls from the upper extremity of a given vertical line AB, and at the same time another perfectly elastic ball B is projected upwards from a horizontal hard plane at the bottom; they meet in some point C, and are reflected back. Determine the point C, so that they may ascend and descend from it continually; and find the velocity of B at that point.

13. Find the fluents of

$$e^x.(\dot{P}+P\dot{x}), \text{ of } \frac{dz^{pn-1}\dot{z}}{(a+cz^n)^m.(e+fz^n)^r}, \text{ and of } \frac{x^{m-1}\dot{x}}{(1-x^m).\sqrt[2m]{(2x^m-1)}}.$$

14. If the middle points of any two edges of a triangular pyramid which do not meet, be joined; shew that the middle point of the connecting line is the centre of gravity of the pyramid.

15. If parallel rays be incident on a spherical surface of a plano-convex glass mirror, whose thickness is a semi-diameter and a half of the spherical surface, prove that they will, after having been refracted at the convex and reflected at the plane surface, converge to that point where the axis intersects the convex surface.

16. Two plane reflectors being inclined to each other at a given angle, determine the diameter of a

circular arc, in which a luminous object must move between them, so that the ray, which has been reflected by any given point of one of them, may after reflection at the second plane, always intersect the arc in the point in which the object is.

17. If the circle of curvature to the vertex of a parabola be described, and another circle touch that, and the arms of a parabola; and so on continually; prove that the radii of the circles will be in the proportion of the numbers, 1, 3, 5, 7, 9, &c.

18. If the force vary according to any law of the distance; shew that in any orbit, at the point where the centripetal and centrifugal forces are equal, the velocity towards the centre of force is a maximum.

19. Determine the nature of the curve by the revolution of which round its axis a solid will be generated; such that a corpuscle placed on its surface will be attracted towards the centre of gravity with a force varying as the distance; the solid revolving round its axis in a given time.

20. Find the horary increment of the area, which the moon, by a radius drawn to the earth, describes in a circular orbit. (Newton, Book III. Prop. 26.)

21. The excentricity of P's orbit (Sect. XI.) being small; find the variation of the major axis in a whole revolution of P, if the force at P be augmented or diminished by a small quantity in the ratio of $1 : 1 \pm n$.

22. If the co-latitude of the place of observation be equal to the moon's declination, or less than it, there will be no inferior or no superior tide, according as the latitude and moon's declination are of the same or different denominations.

23. Let a spherical body descend from rest in a fluid whose specific gravity is to that of the body as $1 : n$. Determine the velocity of the sphere at any point of its descent; and shew that the greatest velocity which it can acquire is equal to that which would be acquired by it in descending from rest, in vacuo, by the constant force of its comparative gravity through a space which is to $\frac{4}{3}$ of its diameter $:: n : 1$.

24. A given cylindrical rod falls by gravity towards a horizontal plane, whilst at the same time its extremity moves freely along the plane. Determine the pressure upon the plane in any position, and the velocity of the moving point.

Tuesday Morning.—Mr. Bland.

FIRST AND SECOND CLASSES.

1. If the terms of the series arising from the expansion of $(a+b)^n$ be multiplied respectively by the terms of the arithmetic series $0x$, $1x$, $2x$, $3x$, &c. find the sum of the resulting series.

2. If a polygon has $(n+4)$ sides; prove that the angles formed at the points of concourse of

these sides produced are together equal to $(2n)$ right angles.

3. If a body move in a curve round a centre of force, and the force by which it is retained in the curve vary in a less ratio than the inverse cube of the distance, prove that the body cannot fall into the centre.

4. Shew that in the spiral, where the angle described by the radius vector $SP \propto SP$ directly; the number of revolutions which have been made by it varies as the square root of the subtangent to the point P.

5. Transform the equation $2x^3 - 2x^2 + 3x + 6 = 0$ into one which shall have its signs alternately positive and negative.

6. Two bodies A and B move in opposite directions with velocities, the sum of which is given. Shew that the sum of the products of each body into the square of its velocity is a *minimum*, when the velocities are reciprocally proportional to the quantities of matter in the bodies.

7. If from one extremity of the diameter of a circle, chords are drawn intersecting the radius which is perpendicular to the diameter or that radius produced, and from the points of intersection ordinates be erected, always equal to the cosine of the arc measured from the opposite extremity of the diameter to the chord; determine the nature of the curve which is the locus of the ordinates.

8. Find the fluent of $\dfrac{d z^{\frac{1}{3}} \dot{z}}{(c^{\frac{5}{3}} - a z^{\frac{2}{3}})^{\frac{3}{5}}}$, and of $\dot{x} \int \dot{x} \int \dot{x} \int \dot{x}$ in infinitum; and find the relation of the abscissa and ordinate of a curve, when $\epsilon^z = \epsilon^x - \epsilon^{-x}$; ϵ being the base of hyp. log., and z and x the arc and abscissa respectively.

9. When parallel rays are incident upon a spherical reflector, shew that the radius of the least circle of aberration varies directly as the cube of the semi-aperture, and inversely as the square of the focal length of the reflector.

10. A particle P is attracted to a sphere by forces $\propto \frac{1}{D^2}$. If on the line joining P and the centre of the sphere a semi-circle be described and made to revolve; it would cut out a portion of the sphere, the attraction to which is equal to the attraction of the remaining part.

11. Find the sum of the series cos. $A + \frac{1}{2}$ cos. $2A + \frac{1}{3}$ cos. $3A + \frac{1}{4}$ cos. $4A$ + &c. and resolve $1 + 5 + 19 + 65 + 211 +$ &c. whose scale of relation is $f - g$, into two geometric series whose corresponding terms added together will give the proposed series.

12. Prove that the mean tides are equally affected by the northerly and southerly declinations of the moon.

13. The quadrant of a circle is impelled by a

stream in its own plane in the direction of the extreme radius. Find the direction in which it will begin to move.

14. Find the equation to the section of a solid generated by the revolution of a given algebraical curve about its axis.

Tuesday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Prove that the opposite sides of an equilateral and equiangular hexagon are parallel.

2. Determine the roots of the equation $x^4 - 4x^3\sqrt[4]{2} + 6x^2\sqrt[4]{4} - 4x\sqrt[4]{8} + 2 = 0$; and find the equation whose roots are $\frac{1}{2}a + \sqrt{(-\frac{3}{4}a^2)}$; $\frac{1}{2}a - \sqrt{(-\frac{3}{4}a^2)}$; $-a$.

3. If two equal forces, inclined at any angle, act upon a body, prove that the compound force bisects that angle.

4. Given the meridian altitudes of the upper and lower limb of the sun $18^0.\ 39'.\ 39''$, and $18^0.\ 7'.\ 3''$. Determine its diameter, and the altitude of its centre.

5. Find the sum of the series,

$6 + 2 - 2 - 6 -$ &c. to 19 terms;

$8 + 20 + 50 + 125$ to 15 terms;

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} +$ &c. to n terms.

6. If a body revolve in a circular orbit about the earth at a distance from its surface equal to 20 radii of the earth; what is the measure of the subtense of the arc described in 1″?

7. If from the extremity of the diameter of a circle a straight line be drawn touching the circle and equal in length to the circumference, and a triangle be formed by joining its extremity and the centre. Prove, that if from any point in this line a perpendicular be drawn to the base, the circumference of the circle described with this as radius, shall be equal to the part of the base intercepted between the perpendicular and the acute angle.

8. The equation $x^4 - \frac{1}{2} x + \frac{3}{16} = 0$ has two equal roots. Find all the roots.

9. Prove that the periodic times of bodies revolving in different ellipses round different centres of force in the foci are in the sesquiplicate ratio of their major axes directly and the subduplicate ratio of the forces inversely.

10. Construct Newton's telescope; and find the magnifying power, and greatest field of view.

11. Find the fluents of

$$\frac{\dot{x}}{x^2\sqrt{(x^2-a^2)}}, \quad \frac{\dot{x}}{x^3-2ax^2+x},$$

where a is less than unity: and the n^{th} fluxion of $\sqrt{x}$.

12. If a body move in a conic section acted upon

by a force tending to the focus S, shew that the velocity at the distance SP is to the velocity at any other distance SQ as a mean proportional between HP and SQ is to a mean proportional between SP and HQ, H being the other focus.

13. Determine the arc of a given circle, the rectangle under whose sine and excess of sine above the cosine is a maximum.

14. The radius of a circle whose area is equal to the surface of a given cone is a mean proportional between the side of the cone and the radius of its base. Required a proof.

15. Compare the absolute forces in the centre and circumference of a circle, so that the periodic times may be the same.

16. An (n^{th}) part of a hollow paraboloid with its vertex downwards is filled with a fluid of known specific gravity; and a sphere of given size and substance is immersed. Find how high the fluid will rise.

Tuesday Afternoon.—Mr. Hustler.

THIRD AND FOURTH CLASSES.

1. If the two sides of a spherical triangle together $= 180^0$, the arc which bisects the vertical angle, bisects the base also.

2. One root of the equation $x^4+x^3-8x^2-16x-8=0$, is $1-\sqrt{(5)}$. Find the other roots

3. Two bodies are projected towards each other in the same vertical plane from two given points, so as to describe the same parabola. Find the point where they meet.

4. Given the right ascension and declination of a star, and the latitude of the place; determine the day of the year when the star rises the same instant with the sun.

5. In the parabola, the normal is the least line which can be drawn from a given point in the axis to the curve.

6. Required the fluxion of $x \times \epsilon^{\tan. x}$, and the following fluents,

$$\int \frac{\dot{x}}{x^3 \sqrt{(1-x^2)}}, \quad \int \frac{b x^{\frac{1}{2}} \dot{x}}{\sqrt{(a^7 - x^7)}}, \quad \int \frac{\dot{x} \text{ h. l. } x}{x}.$$

Shew also that $\int \dot{x}$ cosec. $2x = \frac{1}{2}$ h. l. tan. x.

7. If the bases of a cylinder, and of a cone, have the same radius as a sphere, and each of their altitudes = the diameter of the sphere, the solidity of the cone = the excess of the cylinder above the sphere.

8. A body revolving in an ellipse at the mean distance, is projected perpendicularly to the distance with the velocity with which it is there moving. Shew that it describes a circle, and in the same periodic time in which it would have described the ellipse.

9. The equation to a curve is $y^m = ax^{m-1} + x^m$;

draw its asymptote, and determine the angle which it makes with the axis.

10. Compare the time of emptying a cone and its circumscribing cylinder; the vertex of the cone being downwards and coinciding with the orifice.

11. If an object be seen through a double convex lens, determine the proportion in which it is magnified, when the distances of the eye and object from the center are each equal to half the focal length.

12. Sum the following series,

$$\frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9} \text{ to } n \text{ terms and ad inf.}$$

$$1.4 + 2.5 + 4.6 + \&c. \text{ to } n \text{ terms,}$$

$$\frac{1}{1.2} - \frac{2}{2.3} + \frac{3}{3.4} - \&c. \text{ ad inf.}$$

13. Suppose two bodies fall towards a center of force, one acted upon by a force varying as the distance, and the other by a constant force which is half the variable force at the beginning of the motion. Shew that the velocities acquired at the center will be equal.

14. Give Cotes's construction of the elliptic spiral, and shew in what cases it cuts itself.

15. In the ordinate PN of an ellipse, whose center is C, a point Q is taken so that CQ always $= PN$. What is the curve which passes through the points Q?

16. If a body describes an epicycloid, the force tending to the center of the globe, required the law of the force.

17. If p and q be two weights applied at the circumferences of a wheel and axle, find the proportion between the radii, so that the time of q ascending through a given space may be a minimum, the inertia of the wheel and axle being considered.

Tuesday Evening.—Mr. HUSTLER.

1. A pays P£. to B, on condition that he receives an annuity during the life of an individual, who, according to the tables, may be expected to live n years. What must be the annuity?

2. In an unlimited problem, $mx+ny=p$; if m and n both measure p, m is the least integral value of y, and n of x.

4. A cylinder of indefinite length is placed before a convex reflector, and their axes coincide. Shew that its image is a cone, whose vertex is the principal focus of the reflector.

4. Determine the situation of a fixed star, so that its right ascension may be unaffected by the precession of the equinoxes.

5. Investigate Taylor's Theorem, by the method of differences.

6. A sphere of given radius is suspended in the air. At a given place, day, and hour, determine

the figure of its shadow on a horizontal plane; and shew that the length of the shadow varies as the secant of the sun's zenith distance.

7. Shew that the variation of the radius of curvature of any meridian of the earth, is as the square of the sine of latitude.

8. Prove that the value of a vanishing fraction $\frac{P}{Q}$ may be found by taking successively, if necessary, the first, second, and third, &c. fluxions of the numerator and the denominator; and investigate the method of finding the value of such a fraction when the indices are fractional.

9. *TB*, *BC* are the subtangent and ordinate of a curve whose vertex is *A*, and the tangent of the angle *TCA* is to the tangent of the angle *ACB* in a given ratio. What is the nature of the curve?

10. For any position of the line of the nodes, construct for the inclination of the moon's orbit to the plane of the ecliptic. (Newton, vol. III. Prop. XXXV.)

11. *ABCD* is a section of a four-sided glass prism perpendicular to its axis, having one angle $D = 90^0$, and the opposite angle $B = 135^0$. Shew that a ray of light entering the prism perpendicularly to *AD*, and reflected by *AB*, *BC*, will pass through *CD* without refraction, and thence explain the *Camera lucida*.

12. A paraboloid has its axis parallel to the

horizon, and a flexible chain is wound round its greatest circular section; find the length which will be unwound after t'' have elapsed, a given part being unwound at the beginning.

13. Two equal distances *CA*, *CB* are drawn at right angles to each other from a centre of force *C*. In *CA* any point *D* is taken, and *DE* is drawn to *CB* so that *DE* may equal *CB* or *CA*. Two perfectly elastic balls fall from *A* and *B* at the same time, the force varying as the distance, and are reflected at *D* and *E* by two planes inclined to their motion at $\angle\, 45^0$. Investigate their subsequent motions.

14. Find the fluents of

$$\frac{\dot{x}}{x^5\sqrt{(a^2-x^2)}}, \quad \text{of} \quad \frac{\dot{x}\sqrt{(1+x)}}{(1-x)^{\frac{3}{2}}},$$

$$\text{and of } \frac{\dot{x}\,.\,(1-x^2)}{(1+x^2)\sqrt{(1+ax^2+x^4)}}.$$

15. Find the relation between x and y, when $\ddot{y}\sqrt{(ay)}=\dot{x}^2$; and shew that

$$\int\frac{x^{p-1}\dot{x}}{\sqrt{(1-x^n)^{n-q}}}=\int\frac{x^{q-1}\dot{x}}{\sqrt{(1-x^n)^{n-p}}},$$

between the values of $x=0$, and $x=1$.

16. *ACB* is a quadrant, and one extremity *D* of a line *CD* which equals its chord *AB*, moves along the radius *OB* produced, while the other extremity *C* moves in the periphery of the quadrant. Find the equation and area of the curve described by a point *P* in the middle of the line *CD*.

17. If m be any prime number, and x any other number prime to m. Then x and x^m being severally divided by m, leave the same remainder.

18. Sum the series,

$$1 + \frac{2}{2.3} + \frac{2^2}{3.3^2} + \frac{2^3}{4.3^3} + \text{\&c. ad inf.}$$

$$\frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \text{\&c. to } n \text{ terms},$$

and ad inf. by the method of increments.

19. A paraboloid floats in a fluid, the axis not being perpendicular to the horizon. Determine the position in which it rests; the specific gravities of the paraboloid and the fluid being given.

20. CP, CD, are two semi-conjugate diameters of an ellipse, whose center is C. EP, which is perpendicular to CD, is produced to L making PL equal to CD, and through K the middle point in CL, $MKPN$ is drawn so that KM and KN may each equal CK. Shew that the semi-axes major and minor equal PM and PN respectively, and determine their positions.

21. A person turning up three cards from a common pack, undertakes that the number of points upon them shall be either 29, 19, or 9, reckoning 11 or 1 for the ace, and 10 for each of the court cards. What are the odds against him?

22. If a body A be attracted towards two centres of force S and T, and be projected in a direction oblique to the plane STA, the solids generated by

the motion of the triangle joining *S*, *T*, and the body, shall be proportional to the times of description.

23. *A* and *B* are two ships at sea; *B* moves in a given straight line, and *A* endeavours to overtake *B* by always moving towards it. Having given the velocities of *A* and *B*, investigate the curve traced out by *A*.

1816.

Monday Morning.—Mr. French.

Monday, January 15, 1816.

FIRST AND SECOND CLASSES.

1. Given the logarithms of 6 and 7, shew how the logarithm of 1767 may be computed.

2. If the digits composing any number, be inverted, the difference between the number, so formed, and the original number, is divisible by nine.

3. In the oblique impact of an imperfectly elastic body upon a plane, co-tan. incidence : co-tan. reflection :: force of compression : force of elasticity.

4. In Gregory's telescope, the aberrations, produced by the two reflections, are in the same direction.

5. At a given place, on a given day and hour, the sun's azimuth is double that of a known star; required the distance of the sun from the star.

6. An imperfectly elastic ball being projected from P, a point in the periphery of a circle PQR,

whose center is C, after impinging at Q and R returns to P; required the value of the angle CPQ.

7. Investigate an expression for the length of a caustic by reflection, and apply it, in the case of parallel rays incident upon a concave spherical reflector.

8. In the common pump, given the height of the fixed sucker above the surface of the reservoir, and the space through which the piston descends; required the altitude of the water in the tube after n strokes of the piston.

9. To determine the radius of curvature in the curve, whose ordinate is equal to the circular arc, of which its abscissa is the versed sine.

10. To find the sum of all the powers of the roots of an equation.

11. To draw a diameter to a curve of n dimensions. (M'Laurin's *Algebra.*)

12. Compare the curvatures of the moon's orbit in quadrature and syzygy; supposing that the orbit, independently of the sun's disturbing force, would have been a circle.

13. To determine the number of given points, through which a curve line of the m^{th} order, may be drawn.

Monday Afternoon.—Mr. French.

FIFTH AND SIXTH CLASSES.

1. What decimal of a pound is $11d. \frac{1}{2} \frac{2}{3}$?

2. Investigate the rule for finding the least common multiple of two quantities, and apply it to find the least common multiple of 177 and 2982.

3. Required to express the sum of the alternate terms of a binomial raised to the m^{th} power, beginning with the second.

4. If one side of a triangle be bisected, the sum of the squares of the other two sides is double of the square of half the line bisected, and of the square of the line drawn from the point of bisection to the opposite angle.

5. Compare the area of the hexagon inscribed in a given circle with the area of the circumscribing hexagon.

6. Given the figure of an ellipse, find practically its center and its foci.

7. In an ellipse, if the line Ioi be drawn parallel to the axis minor BCD, and Qoq, parallel to the axis major ACM; then $Io \times oi : Qo \times oq :: BC^2 : AC^2$. Required a proof.

8. Prove, geometrically, that in any plane triangle, the sum of the sides is to their difference as the tangent of half the sum of the angles at the base to the tangent of half their difference.

9. Shew that $(\tan.)^3 60^\circ = 3 \tan. 60^\circ$ to rad. $= 1$.

10. P and W being in equilibrio on an inclined plane, if the whole be put in motion, then P's velocity : W's velocity :: $W : P$.

11. A perfectly hard body is let fall from a given height (a) upon a half-elastic horizontal plane; required the height to which it will rise after impinging the third time upon the plane.

12. Compare the time down $\frac{1}{n}$th part of a given inclined plane with the time down the remainder.

13. Explain the principle of the syphon.

14. Determine the visual angle in Cassegrain's telescope.

15. Given the latitude of the place, and the altitude of the sun in the equinoctial, find the hour-angle.

16. The force tending to the focus of an hyperbola varies inversely as the square of the distance.

17. Required to express the fluxion of the arc in terms of the fluxion of the co-secant.

18. The roots of the equation $ay^3 - by^2 - cy + 1 = 0$, are in harmonic progression, find them.

19. Required the fluents of $\frac{mbx^{-n-1}\dot{x}}{\sqrt{(e+fx^{-n})}}$,

$\frac{\sqrt{(y)}.\dot{y}}{1+y^{\frac{3}{2}}}$, $\frac{\dot{x}}{\sqrt{(x^2-a^2)}}$, $\frac{x^{\frac{1}{2}}\dot{x}}{\sqrt{(2a-x)}}$, $\frac{z\dot{z}}{\sqrt{(a^4-z^4)}}$.

Monday Afternoon.—Mr. Bland.

THIRD AND FOURTH CLASSES.

1. Prove that the sectors of two different circles are equal, when their angles are inversely as the squares of the radii.

2. If a circle be described about the center of gravity of a system of bodies, A, B, C, &c., and any point S be taken in the circumference, shew that $A \times SA^2 + B \times SB^2 + C \times SC^2 +$ &c. is constant.

3. The radius of curvature of the common parabola has to the normal the duplicate ratio that the normal has to the semi-parameter. Required proof.

4. Find the center of gravity of the solid generated by a quadrant of a circle through one-fourth of a revolution about the radius.

5. Given (a) and (b) the $(m)^{\text{th}}$ and $(n)^{\text{th}}$ terms of an arithmetic progression; determine the value of the $(x)^{\text{th}}$ term.

6. Find the fluents of $\dfrac{d\dot{x}}{a^3 - mx^2}$, $\dfrac{d\dot{x}\sqrt{(x)}}{\sqrt{(1-x)}}$,

and $$\frac{x^2\dot{x}}{(x-a)^2 . (x+a)}.$$

7. From a given right cone cut off a parabola such, that its area shall be double the rectangle contained by the segments of the diameter of the base, formed by the section.

8. If the force $\propto \frac{1}{D^2}$, and a body be projected in a given direction with a velocity which is to the velocity in a circle at the same distance in a less ratio than $\sqrt{2} : 1$; determine the nature of the orbit described.

9. If an object be placed between two plane reflectors inclined to each other at any angle, and the eyes of a spectator be in any point between the planes, the distance of the eye from any of the images seen by him, is equal to the length of the path described by the rays which form that image.

10. Prove that no fraction can be represented by a terminating decimal, unless the denominator be 2 or 5, or the product of some powers of 2 and 5.

11. The angle included between the hour-lines of 12 and 3, is equal to the angle included between the hour-lines of 4 and 6, in a horizontal dial. Determine the latitude for which the dial is constructed.

12. What must be the nature of a parabolic curve which revolving round its axis would generate a solid, such that the time of emptying it would be to the time of emptying the circumscribing cylinder in the ratio of 1 : 9.

13. If the attraction of the earth and moon be as their quantities of matter directly and the squares of their distances inversely; what is the nature of the curve in which a body being placed would be equally attracted to both?

14. Compare the ablatitious force with the mean force of P to T (11th Sect.); the periodic times of P and T being as $1 : n$.

15. Prove that if two bodies be projected in similar directions with velocities which are in the subduplicate ratio of the force and distance, they will proceed in similar curves.

Monday Evening.—Mr. Bland.

1. Given the sum (s) and the sum of the squares (S) of a geometric series continued in infinitum. Determine the series.

2. If the bases of two equal cycloids be parallel, and the vertex of one in the base of the other; prove that the angle formed by the intersection of the curves will be a right angle.

3. If two conic sections be described on the same axis major, and have the same abscissæ, the ordinates will be in the sub-duplicate ratio of the *latera recta*. Required proof.

4. In a system of wheels moveable by teeth and pinions, having given the ratios of the number of teeth in each wheel and pinion, determine the number of times the $(n)^{th}$ wheel turns round its axis, while the first performs (m) revolutions.

5. Trace the curve whose equation is $a.(y-b)^2$

$= x.(x-a)^2$, and determine the angle at which the curve cuts the axis when $x=a$.

6. If the altitudes of the sun be taken at the same place on the same day, when he is in the same vertical in opposite directions; shew that the sum of their tangents will be to the sum of their secants, as the sine of the sun's declination is to the sine of the latitude of the place.

7. Prove that the sums of the reciprocals of the $(n)^{\text{th}}$ powers of the odd and even numbers are to each other in the proportion of $2^n - 1 : 1$.

8. Find the sum of the series:

$$\frac{2a+b}{a^2.(a+b)^2} + \frac{2a+3b}{(a+b)^2.(a+2b)^2} + \frac{2a+5b}{(a+2b)^2.(a+3b)^2} + \text{\&c. in inf.}$$

$$\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \text{\&c. to } n \text{ terms by}$$

the method of increments.

and $\frac{1}{4.7.12} + \frac{1}{6.10.16} + \frac{1}{8.13.20} +$ &c. in inf.

9. Determine the inclination of a plane of given length, so that a cylinder of known dimensions and uniform density, may roll freely down it in a given time.

10. A barometer, having some air in the tube, stands at an altitude of (a) inches; but being put under the receiver of an air-pump which contains (n) times as much as its barrel, after m turns it

stands at an altitude (b). Find the standard altitude, and the quantity of air in the tube at first.

11. An aperture of given area is cut from the top to the bottom of the side of a regular vessel full of water. Required its nature and dimensions, such that the velocity of the descending surface may be as the $(n)^{\text{th}}$ power of its distance from the lowest point; the velocity of every particle of the issuing fluid being supposed to vary as the square root of its depth below the surface.

12. Shew that in the spiral where the angle described by the radius vector $SP \propto SP^m$, the areas described by SP in one, two, three, ... n revolutions, measuring from the center, will be as the numbers, 1, 2, 3.... n, raised to the power $\frac{m+2}{m}$.

13. Find the fluents of $\frac{(a+bx).d\dot{x}}{x^3-1}$;

$\frac{\dot{x}}{x^n\sqrt{(a^2-x^2)}}$, where n is an even number;

and $\frac{d\dot{x}}{x.(1+x)^2.(1+x+x^2)}$.

14. Two imperfectly elastic bodies A and B are at a given distance in the same vertical line; A, the higher is acted on by gravity, which is supposed to have no effect on B. Shew that if A fall and strike B successively, the intervals between the strokes decrease in geometric progression; and determine the space passed over after any number of strokes.

15. If a small pencil of diverging rays be incident nearly perpendicularly on the spherical surface of a plane convex glass mirror, the radius of which is known; determine what must be its thickness, so that after refraction at the convex, and reflection at the plane surface, they may converge to that point where the axis intersects the spherical surface; the focus of incidence being given, and at a greater distance from the surface than the diameter of the sphere.

16. Determine the nature of the curve which will refract parallel rays to or from one focus, when the cosine of incidence is to the sine of refraction :: $1 + n : 1$.

17. Given the greatest and least apparent diameters of the moon, find what would be the apparent diameter corresponding to the mean distance; and shew that it is less than the mean apparent diameter.

18. If any number of bodies be retained in horizontal circular orbits by means of strings of unequal lengths, and the distances of the centers from the points of suspension be equal, the times of their revolutions will be the same.

19. Determine the nature and area of a curve such that if a right line be drawn from its vertex making an angle of 45^0 with the axis, the portion of the ordinate intercepted between this line and the curve shall always have to the sub-tangent the

inverse ratio that the ordinate has to the given line (a).

20. Let a sphere of a given diameter be projected in a fluid, the specific gravity of which is to that of the sphere as $1 : n$; having given the velocity of projection, determine what part of it is lost during the time the body describes any given space.

21. Shew that the effect of the sun upon the matter of the earth exterior to the inscribed sphere, to turn it about its center, is equal to the effect which would be produced if one-fifth part of that matter was placed at that point of the earth's equator which is opposite to the sun.

22. Required the area of the rhumb-line considered as a spiral; and shew that its orthographic projection on the plane of the equator is an hyperbolic spiral.

23. If the particles of air be moved from their places by a force which varies according to any given law; it is required to find the law of the force with which they will *continue* to be agitated, supposing the elasticity of the atmosphere to be proportional to its density.

24. If a chain of given weight reaching to the center of the earth be suspended from a cylinder at the surface, round which it is made to wind itself by the descent of a weight (w) unwinding a string supposed to be without weight; determine the

velocity of w at any point, and also where it is the greatest.

Tuesday Morning.—Mr. Bland.

FIRST AND SECOND CLASSES.

1. If a series of arcs be taken in arithmetic progression, the radius of the circle will be to twice the cosine of the common difference as the cosine of any arc taken as a mean is to the sum of the cosines of any two equidistant extremes.

2. Find the sum of a recurring decimal $qppp$ &c. in inf., (q) and (p) containing (m) and (n) digits respectively.

3. In the direct impact of perfectly hard bodies, the difference between the sums of the products of each body into the square of its velocity before and after impact, is equal to the sum of the product of each body into the square of the velocity gained or lost.

4. If S, a, r, n be respectively the sum, first term, common ratio and number of terms of a geometric progression; find the sum of the series,

$$(S+a)+(S+a+ar)+(S+a+ar+ar^2)+\&c.$$

5. Prove that if contiguous and parallel rays of light fall upon a refracting sphere, the homogeneal rays will emerge parallel after n reflections and two refractions, when the least cotemporary variations of

the angles of incidence and refraction are to each other as $n+1 : 1$.

6. If an equilateral triangle be inscribed in a circle, and the adjacent arcs, cut off by two of its sides, be bisected; the line joining the points of bisection will be trisected by the sides.

7. The sum of the distances of a star from two known stars is a minimum, and its declination, which is greater or less than that of each of the others is known; determine its right ascension.

8. Let $y = A + Bx + Cx^2 + Dx^3 + \&c.$ where A, B, C, D, &c. are constant quantities; then if $[y]$, $\left[\frac{\dot{y}}{\dot{x}}\right]$, $\left[\frac{\ddot{y}}{\dot{x}^2}\right]$, &c. be the values of $y, \frac{\dot{y}}{\dot{x}}, \frac{\ddot{y}}{\dot{x}^2}$, &c. when $x=0$; prove that

$$y = [y] + \left[\frac{\dot{y}}{\dot{x}}\right] \cdot \frac{x}{1} + \left[\frac{\ddot{y}}{\dot{x}^2}\right] \cdot \frac{x^2}{1.2} + \left[\frac{\dddot{y}}{\dot{x}^3}\right] \cdot \frac{x^3}{1.2.3} + \&c.$$

9. Find the fluent of $\frac{\dot{z}}{z\sqrt{(a+cz^n)}}$;

and of $\frac{x^p \dot{x}}{(1+x^n)^2}$,

the fluent of $\frac{x^p \dot{x}}{1+x^n}$ being $= A$.

10. The velocity of a body descending from an infinite distance towards a centre of force, is $\frac{1}{n}$th

part of the velocity in a circle at the distance of that point; it is required to determine the law of the force.

11. If P be the place of a comet in its parabolic orbit, and a circle be described through P, the vertex and the focus; shew that the time of moving from perihelion to P will be proportional to the perpendicular drawn from the center of the circle to the axis.

12. If $ay^m\dot{y} = cx^n\dot{y} - ayx^{n-1}\dot{x}$; determine the algebraic relation of x and y.

13. In latitude 45° the mean altitude of the tide is always the same whatever be the declination of the moon.

14. From two bags, one of which contains (m) and the other (n) balls, marked *a, b, c, d,* &c. (m) being greater than (n), two balls are drawn; what is the probability that they have both the same letter?

Tuesday Afternoon.—Mr. Bland.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $a + x + \sqrt{(2ax + x^2)}$.

2. The sum of (n) terms of any arithmetic progression whose common difference is equal to the least term, will be equal to the sum of ($n+1$) mag-

nitudes, each of which is half the greatest term of the progression.

3. If four quantities of the same kind be proportional; the first shall have to the third the same ratio that the second has to the fourth. (Euc. B. 5.)

4. If $A \propto B$, and $B \propto C$; prove that $A \propto mB \pm nC$, where (m) and (n) are known quantities.

5. In the steel-yard, if the weight increase in arithmetic progression, the divisions of the scale will be at equal intervals; and if each of these intervals be equal to the shorter arm, the moveable weight will be equal to the difference of the arithmetic progressionals.

6. Two bodies descend, one vertically through 400 feet, and the other down an inclined plane 500 feet long, and inclined at an angle of 30^0 to the horizon, compare the times of their descents.

7. The solid content of a cone whose base is equal to a great circle of a sphere, and altitude equal to the diameter, is half the solid content of the sphere.

8. If the force $\propto \frac{1}{D^3}$, determine how far a body must fall externally to acquire the velocity in the ellipse.

9. Find the fluents of

$$\frac{(a+bx).\dot{x}}{a^2+x^2}, \quad \text{and} \quad \frac{x^4\dot{x}}{\sqrt{(a^2+x^2)}}.$$

10. Construct Newton's telescope, investigate its magnifying power, and find the linear magnitude of the greatest field of view.

11. If a vertical straight line be placed before a plane mirror inclined at an angle of 45^0 to the horizon, determine the image and its position.

12. Given $\sqrt{\frac{\{(x+a)^3\}}{x-a}}$ = a minimum. Find the value of x.

13. A fluid issuing from the side of a vessel (h) feet high, struck the horizontal plane at a distance of (d) feet from the bottom. Determine the point in the side of the vessel where the orifice is made.

14. If the force $\propto \frac{1}{D^2}$, and bodies fall from different altitudes towards different centers of force, determine the proportions of the times in which they fall through any space.

15. If the force $\propto \frac{bA^m + cA^n}{A^3}$; find the angle between the apsides.

16. If A, A', A'', represent the areas of three similar rectilineal polygons described on the hypothenuse and sides of a right-angled triangle, $A = A' + A''$. Required a proof.

Tuesday Afternoon.—Mr. French.

THIRD AND FOURTH CLASSES.

1. Prove, geometrically, that $1 + \cos. 2\theta = 2\cos.^2 \theta$.

2. A perfectly elastic ball, projected from A directly up an inclined plane, AB, strikes a vertical plane passing through B and returns to A; required its velocity at B, the length of the plane being 36 feet, and its elevation 30°.

3. In an hemispheroid emptying itself by a small orifice in the vertex, compare the time in which the surface of the fluid descends through the upper half of its axis, with the time through the lower.

4. To find the variation of the angle, which a given object subtends at the eye when viewed through a convex lens, the object being farther from the lens than its principal focus (F), and the eye nearer to the lens than its principal focus (f).

5. The precession in right ascension is positive when the angle of position is acute, and negative when it is obtuse. Required a proof.

6. The least error in time due to a given error in the altitude of a known star being b''; to determine the latitude of the place, and the true zenith-distance of the star.

7. To find the area of the conchoid of Nicomedes.

8. If the moveable orbit be projected *in antecedentia*, with a velocity equal to that of P *in consequentia*, shew that the velocity of p vanishes, when Cp becomes the least distance in the ellipse.

9. Investigate the equation between the perpendicular and the distance, in the lituus $\left(\angle \propto \frac{1}{\text{dist.}^2}\right)$ and determine the point of contrary flexure in the curve.

10. The roots of the equation, $y^4 - 8y^3 + 14y^2 + 8y - 15 = 0$, are in arithmetical progression. Find them.

11. Required the fluxion of the arc whose sine $= 2y\sqrt{(1-y^2)}$.

12. Apply Newton's method of making a body oscillate in a cycloid, to the common cycloid.

13. Find the following fluents:

$$\int \frac{x^4 \dot{x}}{\sqrt{(a^2 - x^2)}}, \quad \int \frac{2a\dot{x}}{x\sqrt{(a^3 + x^3)}}, \int \dot{\theta} \cos.^3 \theta.$$

14. Sum the following series:

$$\frac{1}{1.3} - \frac{1}{3.5} \text{ \&c. in inf.}$$

$$1 + 3 + 7 + 15 + \text{ \&c. to } n \text{ terms.}$$

$$\frac{1}{1.5.9} + \frac{1}{5.9.13} + \text{ \&c. to } n \text{ terms by increments.}$$

Tuesday Evening.—Mr. French.

1. If a sum p, at compound interest, in n years amounts to m, in what time will the same sum amount to M, at the same rate?

2. A cylindrical vessel, of given thickness, is required to be of a certain capacity, find the least quantity of material with which it can be made.

3. A thin rod, formed of the arcs of two entire, unequal cycloids, lying in the same plane, and on opposite sides of the line of their bases, floats upon a fluid, sinking to the point at which the arcs are united; to determine its position when at rest.

4. To find the surface of a sphere by the method of indivisibles.

5. Required the time of oscillation in a finite circular arc.

6. To determine that point in the periphery of an ellipse, at which the angle contained between the normal and the distance from the center, is a maximum.

7. The place of the earth, when a star's aberration in declination $= 0$, being d_0; and A_0 (lying to the eastward of the point of syzygy) being the place of the aberratic point, when its aberration in right ascension $= 0$; required the position of d_0 with respect to A_0.

8. Solve the following equation $x^3 - 9x + 28 = 0$ by a process similar to that employed by Cardan.

9. If a small pencil of parallel rays fall upon a concave spherical surface, and every ray be reflected, the density of the incident pencil is to the density (supposed uniform) of rays in the least circle of aberration, as the area of the circle, whose diameter is the versed sine of the aperture, to the whole surface of the sphere, very nearly; required a proof.

10. Find the n roots of unity, and shew how the quadratics combine, when n is an odd number.

11. The weight of a given globe being inconsiderable, when compared with the weight of an equal bulk of fluid, prove that, in its ascent, the velocity is uniform, and equal to the velocity by gravity through $\frac{4}{3}$ds of its diameter.

12. Investigate an expression for the pressure on the axis of a mechanical power in motion, and apply it in the case of the single moveable pulley.

13. Sum the following series:

$$\frac{11}{1.2.3.4} + \frac{17}{4.5.6.7} + \frac{23}{7.8.9.10} + \text{\&c. to } n \text{ terms.}$$

$$\frac{5}{1.2.3} \cdot \frac{1}{2^2} + \frac{6}{2.3.4} \cdot \frac{1}{2^3} + \text{\&c. to } n \text{ terms.}$$

$$\frac{1}{1.2} - \frac{1}{4.5} + \frac{1}{7.8} - \text{\&c. in infinitum.}$$

14. Explain D'Alembert's *principle*; and apply it to find the accelerating force on a body drawn up

an inclined plane, by the action of a power parallel to the plane.

15. In any square number, 4 is the only digit which can occupy both the units and tens places.

16. To find the whole number of equal and regular figures, which may be described upon the surface of a sphere so as exactly to cover it.

17. *BM* is a chord of a circle, whose center is *C*, and *CEF* any radius cutting *BM* in the point *E*; at every point *E*, *EP* is erected perpendicular to *BM* and equal to *EF*; required the locus of *P*.

18. In the catenary, the horizontal tension is the same at every point, to determine its actual value.

19. Find the following fluents:

$$\int \frac{x\dot{x}}{(1+x)^3.(1+x+x^2)^{\frac{1}{2}}}, \quad \int b.x^{\frac{3}{2}}\dot{x}.\frac{\sqrt{(2a-x)}}{(a-x)^2},$$

$$\int az^2.y^{n-2}\dot{y}, \text{ if } \dot{z}=(b+cy^n)^m.\dot{y}.$$

20. A body, acted upon by gravity, is projected horizontally, with a given velocity, along the interior surface of a cylinder; required to trace its path upon the surface of the cylinder.

21. To calculate the probability of throwing two assigned numbers, *A* and *B*, with *m* dice, in *n* throws.

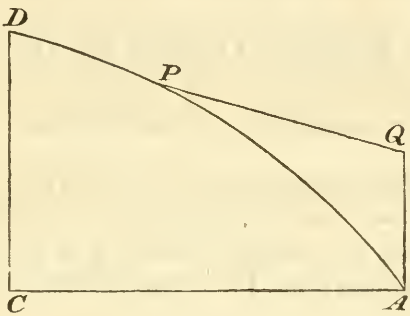
22. Solve the following fluxional equations:

$$(x-y).\dot{x}\dot{y}=y.\dot{x}^2+(a-x).\dot{y}^2,$$

and $\frac{\ddot{x}}{\dot{z}^2}+x+\cos. mz=0.$

23. If an equilateral polygon of 2^n sides be inscribed in a circle, whose rad. $=1$, the value of each side is $\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2 \&c.}}}}$ where the numeral 2 is repeated $(n-1)$ times.

24. ADC is a common parabola, AQ a tangent at the vertex, and PQ a tangent at P, meeting it in Q; to determine the point P, such, that the resistance



on the solid, generated by the revolution of $DPQA$ about CA, when moving in the direction of its axis, may be the least possible.

1817.

Monday Morning.—Mr. Peacock.

MONDAY, JANUARY 13, 1817.

FIRST AND SECOND CLASSES.

1. WHAT decimal of £1. is 3*s*. $3\frac{3}{4}$*d*.?

2. Find the integer values of x and y, which satisfy the equation $13x+14y=200$.

3. Prove that

$$\theta = \tan.\theta - \frac{1}{3}\tan.\theta^3 + \frac{1}{5}\tan.\theta^5 - \&c.$$

† 4. Find the integral of $\frac{dx}{1+x^3}$. *A very memorable problem*

6. Explain what is meant by the *particular solutions* of differential equations. Give an instance in the equation

$$y\,dx - x\,dy = n\sqrt{\{dx^2+dy^2\}}.$$

6. Find expressions for the range and time of flight of a body projected from a given point *above* a given plane.

† *The first official recognition of the continental school of Mathematicians at Cambridge dates from nine o'clock in the morning of Monday January 13. 1817, when Peacock put into the hands of each Candidate for honours a printed paper the fourth question of which stands thus Find the integral of $\frac{dx}{1+x^3}$. Athenaeum Nov. 20. 1858 p. 649.*

7. Two vessels filled with air of different densities communicate by a tube. Find the velocity with which the air will rush into the vessel containing the rarer air.

8. Explain the causes of the following lunar inequalities: (1.) The evection.
(2.) The variation.
(3.) The annual equation.

9. The aberration which arises from the spherical surfaces of lenses is very small, compared with that which is caused by the unequal refrangibility of light.

10. Prove that in the calculus of variations,

$$\delta dv = d\delta v,$$

v being a function of x.

Monday Afternoon.—Mr. PEACOCK.

FIFTH AND SIXTH CLASSES.

1. Find the quotient of 75.04 divided by 3.02101 to three places of decimals.

2. Find the amount of £70. in three years, at $3\frac{1}{2}$ per cent. allowing simple interest.

3. Shew that

$$\{\sqrt{(a+b\sqrt{-1})} + \sqrt{(a-b\sqrt{-1})}\}^2$$
$$= 2a + 2\sqrt{\{a^2+b^2\}}.$$

4. The number N is divisible by 7, if $a_0+a_1.3+a_2.3^2+a_3.3^3+$ &c. $+a_n.3^n$, be divisible by 7; a_0, a_1, a_2, &c. being the digits of the number, reckoning from the place of units.

5. The middle term of the expansion of $(1+x)^n$, when n is even, is

$$=2^{\frac{n}{2}}.\frac{1.3.5....(n-1)}{1.2.3....\left(\frac{n}{2}\right)}.x^{\frac{n}{2}}$$

6. Explain the method of constructing a table of sines.

7. Solve the equation

$$x^4+px^3+qx^2+px+1=0.$$

8. There are at least as many impossible roots in the original equation as in the equation of limits.

9. Explain what is meant by the modulus of a system of logarithms, and shew how it is determined.

10. If $\pi=3.14159$, $t=$time of oscillation, and l the length of the pendulum, then

$$t=\pi\sqrt{\frac{l}{2mf}}.$$

11. Explain the method of determining the right ascensions of the fixed stars.

12. Enumerate and explain the phænomena exhibited by the moon in the course of a month.

13. Explain the method of determining the specific gravities of bodies.

14. A given rectilinear object is placed at a given point in the axis of a concave mirror. Required the nature and position of its image.

15. Find the fluxion of sin. x.

16. Find the values of x, which make the function $x^3 - x^2 - 8x + 12$ a *maximum* or a *minimum*.

17. Expand $(1+x)^{\frac{1}{2}}$ by means of Maclaurin's theorem.

Monday Afternoon.—Mr. White.

THIRD AND FOURTH CLASSES.

1. A sum P is due at the end of m years; find the difference between its amount at the end of $(m+n)$ years, and the amount of its present value at the end of $(m+n)$ years, at simple interest.

2. It is required to find two harmonic means between 3 and 12.

3. If two circles intersect each other in A, B; any two parallel lines CD, EF, drawn through A, B, respectively, and cutting the circles in C, D; E, F; are equal. Required a proof.

4. An object is placed before a concave spherical

reflector. Required its position, when the image is inverted, and equal to twice the object.

5. It is required to find the principal focus of a concavo-convex lens, of a rarer medium, whose thickness is inconsiderable.

6. A body falls down a given length of an inclined plane, and impinging upon the horizontal plane moves along it; required the elevation of the plane, when the time of moving upon the horizontal plane, over a space equal to the height of the plane, is equal to the time down the plane.

7. Prove that when a body oscillates in a cycloid, the whole force which stretches the string, varies as the length of that part of it, which is not in contact with the upper cycloid.

8. The length of the shadow of an upright rod at noon on the shortest day : its length at noon on the longest day :: $n : 1$. Prove that the sine of twice the latitude : the sine of twice the obliquity :: $(n+1) : (n-1)$.

9. A hemispherical vessel standing upon its base is filled with fluid; compare the pressures perpendicular to its plane and curved surfaces.

10. Compare the times of emptying a vessel in the form of a parabolic frustum, by a small orifice in its base, when it is placed with the vertex of the parabola downwards, and when it is placed with the vertex upwards.

11. Compare the time of descent in a given logarithmic spiral, to the center S, from a given point P, with the periodic time in a circle, at the distance SP.

12. If a body fall down the radius of a circle, F varying as $(\text{dist.})^3$, and ascend on the other side through radius by a repulsive force; shew that it will acquire the velocity of revolution in the circle.

13. In any spherical triangle whose sides are a, b, c, and opposite angles A, B, C; if $b=c$, shew that

$$\sin. b = \frac{\sin. \frac{a}{2}}{\sin. \frac{A}{2}}, \text{ and } \sin. B = \frac{\cos. \frac{A}{2}}{\cos. \frac{a}{2}}.$$

14. Sum the series

$$\frac{3}{1.2.2} + \frac{4}{2.3.2^2} + \frac{5}{3.4.2^3} + \&\text{c. to } n \text{ terms},$$

$$\text{and } \frac{1}{1.5} + \frac{1}{3.7} + \frac{1}{5.9} + \&\text{c. to } n \text{ terms}.$$

15. Find the fluents of

$$\frac{p\dot{x}}{\surd(x).\surd(a-bx)}, \text{ and } \frac{\dot{x}}{x\surd(1+\sqrt{x})}.$$

16. Find the centre of gyration of the plane of a semicircle, revolving about its diameter.

Monday Evening.—Mr. White.

1. The present value of a freehold estate of £100. *per annum*, subject to the payment of a certain sum (A) at the end of every two years, is £1000. allowing 5 *per cent.* compound interest. Required to determine the sum A.

2. $1^3=1$, $2^3=3+5$, $3^3=7+9+11$, $4^3=13+15+17+19$, &c.=&c. Prove the formula for n^3.

3. Two equal hard bodies are projected at the same instant towards each other, from the two extremities of a vertical line, each with the velocity which would be acquired by falling down it. Required the interval of time, between their impact and their arrival at the lower extremity of the line.

4. A hemispherical vessel, of given weight, floats upon a fluid, with one third of its axis below the surface; required the weight which must be put into it, so that it may float with two-thirds of its axis below the surface.

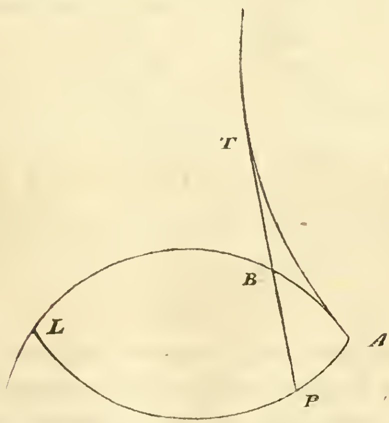
5. A ray of light passes through a prism of a denser medium, and the ray within makes two acute angles with the sides of the prism; if I, i, be the angles which the incident and emergent rays make with the perpendiculars to the surfaces, and R, r, the angles which the ray within makes with the same perpendiculars, prove that when the deviation is a minimum, $I=i$, and $R=r$.

6. If the moon and sun be upon the meridian at the same instant, and $A, a,$ be the increases of their right-ascensions (supposed uniform) in one solar day; $A, a,$ being reckoned in time at 15° to one hour; shew that the exact interval between their next following transits $= \frac{(A-a)}{24-(A-a)} \times 24$ hours, solar time.

7. Prove that by means of the series of weights 1, 2, 4, 8, 16, &c. any weight not exceeding the sum of the weights, can be weighed.

8. Prove that a circular arc of given radius, will oscillate through a given angle, in its own plane, about its middle point, in the same time, whatever be its length.

9. When a body oscillates in a hypocycloid, as in



the tenth section; if TBP be any position of the string, shew that the time of describing AP : the time of describing PL :: the arc AB : the arc BL.

10. Prove that the roots of the equation, $x^m + 1 = 0$, when m is an odd number, are, $\frac{1}{r^{m-2}}$ .... $\frac{1}{r^3}$, $\frac{1}{r}$, r, r^3, ... r^{m-2}, r^m, where $r = \left(\cos. \frac{\pi}{m} + \sqrt{(-1)} \sin. \frac{\pi}{m}\right)$ and $r^m = -1$.

11. Prove that a regular octahedron inscribed in a sphere : the cube of the radius :: 4 : 3.

12. A body is projected, at a given distance r, at an angle of 30^0, with the velocity acquired from infinity. Find the time elapsed when the body is at the distance $\frac{r}{2}$ from the center; supposing the force to vary as $\frac{1}{(\text{dist.})^2}$, and to be equal to twice the force of gravity at the point of projection. (Newt. Sect. 8.)

13. The sides of a spherical triangle are a, b, c; and the opposite angles A, B, C; if A and C be invariable, and b be increased by a small quantity, shew that a will be increased or diminished, according as c is less or greater than a quadrant.

14. The mean motion of the nodes of the fourth satellite of Jupiter, caused by the disturbing action of the third, ought, according to the principles of the eleventh section, to be regressive; whilst this

regression takes place, can the node of the orbit of the fourth satellite be progressive upon Jupiter's orbit?

15. When rays diverge, from a point beyond it's principal focus, upon a double convex lens of a denser medium; if q' be the distance of the focus of refracted rays from the second surface, the thickness (t) being small; and q that distance when t is neglected; shew that $\frac{1}{q'} = \frac{1}{q} + \frac{(nd-r)^2}{(1+n)d^2r^2} \cdot t$, nearly; where r is the radius of the first surface, d the distance of the focus of incidence from it, and $1+n : 1 :: \sin. I : \sin. R$.

16. A ball, whose elasticity : perfect elasticity :: $n : 1$, is projected obliquely upwards, from a point in the horizontal plane, upon which it impinges and rebounds continually; prove that the ranges and times of flight in the successive parabolas described, form geometric progressions; and find their sum.

17. If the resistance vary as (vel.)^3, and a body fall by the action of a constant force; find the time in which it will acquire a given velocity.

18. Find the fluent of $\frac{\dot{x}}{(a^2+x^2)^{\frac{2r+1}{2}}}$, and in the fluent of $\frac{x^m \dot{x}}{x^n - px^{(n-1)} + qx^{(n-2)} - \&c.}$, m being greater than n, shew that the coefficients of all the terms which involve higher powers of x than the $(m-n+1)^{\text{th}}$ will vanish.

19. Sum the following series,

$$\frac{1}{1.3.3} - \frac{2}{3.5.3^2} + \frac{3}{5.7.3^3} - \text{\&c. ad infinitum.}$$

also, $\frac{2}{1.3.3} + \frac{3}{3.5.3^2} + \frac{4}{5.7.3^3} + $ &c. to n terms by the method of increments.

20. An isosceles right-angled triangle is immersed in a fluid, having one of its sides coincident with the surface; find the distance of the center of pressure from the side immersed.

21. (Sect. XI. Prop. 66.) Cor. 14. If ST and the absolute force of S be changed, the periodic linear errors of $P \propto \frac{1}{(\text{per}^{c}.\text{ time of } T)^2}$. Cor. 15. If ST, PT, be changed in the same proportion, and also the absolute forces of S and T be changed in the same proportion, the periodic linear errors of P vary as PT. Required proof: and hence to compare the periodic linear errors of P in different systems of S, T, P, where the form and inclination only of the orbits remain the same.

22. If out of 86 persons born, one dies at the end of every year; and m, n, be the complements to 86 of the ages of two individuals A, B, m being less than n, prove that the probability of A's surviving $B = \frac{m-1}{2n}$.

23. In a system of two pulleys, where each string is attached to the weight, P draws up W; find the

accelerating force on P, the tensions of the strings, and the pressures upon the centers of the pulleys; taking into consideration the weight and inertia of the pulleys.

Tuesday Morning.—Mr. White.

FIRST AND SECOND CLASSES.

1. The logarithm of 37852 is 4.5787767; the logarithm of 37853 is 4.5787882; required the number, corresponding to the logarithm 6.5787836.

2. If the sides of a spherical triangle, AB, AC, be produced to b, c, so that Bb, Cc, shall be the semi-supplements of AB, AC, respectively; prove that the arc bc will subtend an angle at the center of the sphere, equal to the angle between the chords of AB, AC.

3. If the radii, of the tube, and of the basin, of a barometer, be 1 and 3; and the index shews, at sight, the height of the mercury in the tube, above that in the basin; prove that the inch upon the scale : a real inch :: 8 : 9, the thickness of the tube being neglected.

4. The altitudes of a circum-polar star are observed at two instants, when it has the same azimuth, before it passes the meridian; and also the time between those instants; from these data, determine the latitude of the place.

5. Having given the distance at which a short-sighted person can see distinctly, it is required, to find the distance between a given object-glass, and given eye-glass, in an astronomical telescope, when adapted to such an eye, and to distant objects.

6. The periodic times of the first and second satellites of Jupiter, are, ($1^d. 18^h. 27^m. 33^s.$) and ($3^d. 13^h. 13^m. 42^s.$) If a, a', be their mean distances, prove that, $a : a' :: 1 : 2^{\frac{2}{3}} \times \left(1 + \frac{2}{3} \times \frac{1}{274}\right)$ nearly.

7. If the $\sqrt[3]{\tan. \left(45 - \frac{z}{2}\right)} = \tan. u.$ prove that,
$\sqrt[3]{(\tan. z + \sec. z)} + \sqrt[3]{(\tan. z - \sec. z)} = 2 \cot. 2u.$

8. Determine the weights which must be selected out of the series, 1, 2, 4, 8, &c. pounds, in order to weigh 1317 pounds.

9. If a body be projected obliquely upwards, shew that the square of its velocity, will always be equal to the square of the velocity of projection, diminished by the square of the velocity which it would acquire by falling down its perpendicular height, above the horizontal plane passing through the point of projection.

10. A body describes a circle, the center of force being in the circumference; another body describes an equal circle, the center of force being in the center of the circle, and the absolute force being

one-fourth of its former value. Compare the times in which the circles are described.

11. Prove, that $(a+b)^n = a^n + n.a^n.\frac{b}{(a+b)}$ $+ n.\frac{n+1}{2}.a^n.\frac{b^2}{(a+b)^2} + n.\frac{n+1}{2}.\frac{n+2}{3}.a^n.\frac{b^3}{(a+b)^3}$ + &c. and by this theorem, sum the series,

$\frac{1}{3}+\frac{3}{2}.\frac{1}{3^2}+\frac{3.4}{2.3}.\frac{1}{3^3}+\frac{3.4.5}{2.3.4}.\frac{1}{3^4}$ + &c. ad infinitum.

12. Upon one side of the given straight line *AB* describe a semicircle, and upon the other side an equilateral triangle *ADB*; if a solid be generated by the revolution of this figure about *DC*, *C* being the center of the semicircle; prove that it will rest upon the horizontal plane, upon any point of it's spherical surface.

Tuesday Afternoon.—Mr. White.

FIFTH AND SIXTH CLASSES.

1. It is required to express 23°. 27′. 53″. in hours, minutes, and seconds.

2. Find the discount upon £125. 10*s.* 0*d.* payable at the end of three years, at 4½ *per cent.* simple interest.

3. It is required to determine the point *C* in the semicircle *ACB*, such that the three sides of the triangle *ACB* shall be in geometrical progression.

4. Two bodies 1, 2, moving with velocities 1, 2, whose elasticity : perfect elasticity :: 1 : 2, impinge upon each other, making the angles of 30^0, and 90^0, respectively, with the plane touching them at the point of contact. Required the directions in which they will move, and their velocities after impact.

5. A body is projected down an inclined plane, with the velocity acquired in falling down its height, and describes the length of the plane in the time of falling down its height. Required the elevation of the plane.

6. In a quadrantal triangle, the angle opposite the quadrant, and one of the other angles, are given; find the remaining angle.

7. Prove that the illumined phase of Mars is the least, when he is in quadrature.

8. If an object be viewed through a glass plate of given thickness, determine how much the apparent distance is less than the true.

9. It is required to determine the brightest part of the visible area in Galileo's telescope.

10. A circle and its inscribed hexagon, move with equal velocities, in directions inclined at angles of 30^0 and 60^0, respectively, to their planes. Compare the resistances perpendicular to their motions.

11. Sum the following series to n terms,

$$r - \frac{r^{\frac{3}{2}}}{2} + \frac{r^2}{4} - \&c.$$

and $1.2 + 2.5 + 3.8 + 4.11 + \&c.$

12. Find the fluents of

$$\frac{p\dot{x}}{a - bx^2}, \text{ and } \frac{x^3\dot{x}}{\sqrt{(a-x)}}.$$

13. Having given the ratio of the periodic times in two circles, described about different centers of force situated in their centers, and also the ratio of the radii, it is required to find the ratio of the absolute forces.

14. Determine the angle between the apsides in an orbit nearly circular, the force being constant; taking an ellipse about the center for the revolving orbit.

Tuesday Afternoon.—Mr. Peacock.

THIRD AND FOURTH CLASSES.

1. If $\frac{p_0}{q_0}$ and $\frac{p_1}{q_1}$ be two consecutive terms in a series of fractions converging towards $\frac{a}{b}$; then

$$p_0 q_1 - p_1 q_0 = \pm 1.$$

2. Explain what is meant by the *conjugate points* of curve lines.

3. If $u = f\{x, y\}$, shew that

$$\frac{d^2u}{dxdy} = \frac{du^2}{dydx}.$$

4. Find the integral or fluent of

$$\frac{dx}{x^3 - 7x^2 + 12x}.$$

5. If a body be projected perpendicularly upwards with a velocity (a), its height (x), at the end of the time (t), is determined from the equation

$$(a - 2mt)^2 = 4m\left(\frac{a^2}{4m} - x\right).$$

6. Enumerate the different practical methods of determining the latitude of a ship at sea.

7. Explain the method of measuring altitudes, by means of the barometer and thermometer.

8. A given rectilinear object is placed before a spherical reflector of given radius. Find the equation to the conic section which is its image.

9. Find an expression for the whole time of descent of a body from a distance (a) to the center of force, when the force varies inversely as the square of the distance.

10. Mention some of the problems, upon which the trisection of an angle, by common geometry, may be made to depend.

Tuesday Evening.—Mr. Peacock.

1. Demonstrate the rule for the extraction of the square root in numbers.

2. Every prime number of the form $4n+1$ is the sum of two squares.

3. Approximate to the value of x in the equation

$$x^3 - 2x - 5 = 0,$$

and explain the defects of the methods of approximation, as given by Newton and Ralphson.

4. Prove that

$$\Delta^n u_x = u_{x+n} - \frac{n}{1} \cdot u_{x+n-1} + \frac{n(n-1)}{1 . 2} u_{x+n-2} - \&c.$$

5. Integrate the differentials

$$\frac{dx}{x^5\sqrt{\{1+x^2\}}}, \quad \frac{a^x dx}{x^2}, \text{ and } dx \cos.^2 x \sin.^3 x.$$

6. Integrate the differences or increments

$$x^3 \text{ and } e^x \cos. x\,\theta.$$

7. Integrate the different equations

(1.) $\dfrac{d^2 y}{d x^2} = \dfrac{m}{(a-y)^2}$.

(2.) $d^2 y + Aydx^2 = Xdx^2$, where X is a function of x.

(3.) $\dfrac{dz}{dx} - \dfrac{y}{x} \cdot \dfrac{dz}{dy} = -\dfrac{y^2}{x^2}$.

8. Integrate the equation of differences,

$$u_{x+2} - Au_{x+1} + Bu_x = 0.$$

9. Given the length of the curve; required its nature when its centre of gravity is most remote from the axis.

10. If two lines intersect each other within a parabola, the ratio of the rectangles contained by their respective segments will be the same with the ratio of the rectangles made by the segments of any other two lines which intersect each other, and which are respectively parallel to the former.

11. Apply D'Alembert's principle to the determination of the distance of the centres of oscillation and suspension in a compound pendulum.

12. A triangular prism being immersed in a fluid of greater specific gravity than itself, it is required to determine the different positions in which it will rest in equilibrium.

13. A machine, driven by the impulse of a stream, produces the greatest effect when the wheel moves with one-third of the velocity of the water.

14. At a place whose latitude is $48^\circ. 50'. 14''$, the meridian altitude of the sun's upper limb was observed to be $62^\circ. 29'. 56''$; it is required to determine the sun's declination, the refraction being $29''$, the sun's parallax and apparent diameter of their mean values, and the sine of $27^\circ. 30'. 4'' = .4617$.

15. Explain the method of correcting an error in the longitude of a place, by means of the occultation of a given fixed star by the moon.

16. If r be the radius of an isosceles lens, whose focal length is equal to that of a lens whose radii are r_1 and r_2; then

$$\frac{1}{r} = \frac{1}{2}\left\{\frac{1}{r_1} + \frac{1}{r_2}\right\}.$$

17. If D be the length of a degree of the meridian at a point whose latitude is λ, Δ the length of a degree of a curve perpendicular to the meridian at that point, a the axis major of the meridian, and e the difference of the semi-axes; then

$$\frac{e}{a} = \frac{\Delta - D}{2\,\Delta\cos.^2\lambda} \text{ (nearly)}.$$

18. The moon is retained in her orbit by the force of gravity. Newton. Lib. III. Prop. 4.

19. The sum of the sides of a right-angled triangle remaining the same, required the nature of the curve to which the hypothenuse is always a tangent.

20. Explain the method of drawing a normal to a given curve surface.

21. Give an account of the controversy between the followers of Newton and Leibnitz, concerning the measure of motion; and reconcile the experiments and results to which the latter appealed, with the measure assumed by the former.

22. If two chords of a circle intersect each other at right angles, the sum of the squares described upon the four segments is equal to the square described upon the diameter.

23. Give some account of the analysis of the ancient geometers. Exemplify it in the solution of the following problem: "To bisect a triangle by a straight line drawn through a given point in one of its sides."

1818.

Monday Morning —Mr. French.

Monday, January 20, 1818.

FIRST AND SECOND CLASSES.

1. The present value of an annuity, to continue for a term of years at a given rate of compound interest, $= m \times$ the present value of the same annuity, to be paid only during the latter half of the same term; required to find when the annuity will cease.

2. To determine the numerical value of the arc A which will *satisfy* the following equation:

$$\sin. B + \sin. (A - B) + \sin. (2A + B) = \sin. (A + B) + \sin. (2A - B).$$

3. Prove that the sum of all the coefficients of a binomial raised to the $(2n)^{\text{th}}$ power : the coefficient of its middle term :: 2 . 4 . 6, &c. to n factors : 1 . 3 . 5 . &c. to n factors.

4. A body is suspended from a given point in the horizontal plane, by a string of known length, which is thrust out of its vertical position by a rod

(supposed without weight) acting, from a given point in the plane, against the body; shew that the tension of the string varies inversely as the tangent of the inclination of the rod to the horizon.

5. Two equal hollow paraboloids have a common axis, which is vertical, and such a quantity of water is poured in between them, as just to touch the lowest point of the inner figure; demonstrate that the surface of the water will be a tangent plane to this figure, in any position of the common axis.

6. In Gregory's telescope, the focal length of the larger reflector, the position and focal length of the eye-glass, and the distance between the two images of a remote object being given; required to find the position and focal length of the smaller reflector, which will cause the telescope to magnify the object in any proposed ratio.

7. Having given $nt = u - e . \sin. u$; required the first four terms of the series expressing u in terms of nt.

8. A spherical body descends in a fluid by gravity; to determine the quantity of the resistance, when the body has described a given space.

9. The force varying inversely as (dist.)[illegible] and the velocity being that which would be acquired from infinity, a body is projected from an apse; compare the time of its descent to the center, with the periodic time in a circle, whose radius = half the apsidal distance.

10. To find the fluent of $\frac{\dot{x}\sqrt{(1-x^2)}}{(1+x)^2}$.

11. Sum the following series:

$$\frac{1}{1.1}+\frac{1}{3.5}+\frac{1}{5.9}+\text{ \&c. in inf.}$$

12. A body describes a parabola about the focus, and at the same time the figure moves uniformly in a direction *perpendicular* to its axis, which continues parallel to itself; to determine the path described by the body in fixed space.

Monday Afternoon.—Mr. French.

FIFTH AND SIXTH CLASSES.

1. $\frac{x}{x+2}-\frac{x-9}{3x-20}=\frac{9}{13}$. Required the values of x.

2. Required the ratio which is one half of the ratio ($\sqrt{32}$: 25.)

3. The sum of an arithmetic series is 5, the first term 11, and the common difference -5; find the number of terms.

4. To determine the value of tan. 30^0 to two places of decimals. (rad. = 10000.)

5. P being any point in an ellipse, whose semi-axis major is AC; prove, that, if the normal (PG)

be produced to meet the conjugate diameter in F and the minor axis in V,

$$PF.PV=(AC)^2.$$

6. Two circles touch each other internally, and the area of the *lune* cut out of the larger is equal to twice the area of the smaller circle. Required the ratio of the diameters of these circles.

7. A body is projected perpendicularly upward with a velocity of 64 feet per second; find the time of ascent through 63 feet.

8. The length of the gage of a condenser is 12 inches, and the space occupied by the air in it, after two descents of the sucker, is half its whole length; to determine the space which the air will occupy after the third descent of the sucker.

9. Having given the focus of incidence of a pencil of rays which passes nearly perpendicularly through the sides of a prism, and sin. I : sin. R :: n : 1 out of the ambient medium into the prism; required (n being a proper fraction) to find the focus of emergent rays.

10. If a star be situated nearer to the pole of the ecliptic than to that of the equinoctial, shew that its right ascension exceeds 180^0.

11. A body is revolving in a given circle about its center, if the absolute central force be increased in a given ratio, what change must be made in the

velocity of the body, that it may still describe the same circle?

12. Demonstrate, as Newton has done (Cor. 2. Prop. 10.), that the periodic times in all ellipses about the same center are equal.

13. Assuming the velocity to vary as $\frac{\sqrt{(a-x)}}{\sqrt{(x)}}$, a being the initial distance and x the variable distance of the body from the center of force; to determine the law of the centripetal force.

14. One root of the equation $(x^3-11x^2+37x-35=0)$ is $3+\sqrt{2}$; required the remaining roots.

15. Required the fluxion of $\frac{y\sqrt{(a^2-y^2)}}{a^3-y^3}$.

16. Find the fluent of $\frac{az^2\dot{z}}{e-fz}$.

Monday Afternoon.—Mr. Fallows.

THIRD AND FOURTH CLASSES.

1. What part of half a crown is equal to $\frac{3}{7}$ of 1*s.* 5½*d.*

2. Of all triangles under a given perimeter and a determinate side, shew that to be the greatest in which the two indeterminate sides are equal.

3. If the p^{th} and q^{th} terms of an arithmetical progression be P and Q, find the sum of n terms of the series.

4. Transform the cubic equation $x^3+px^2+qx+r=0$, whose roots are a, b, c, into another whose roots are

$$\left(\frac{1}{a^2}+\frac{1}{b^2}\right),\quad \left(\frac{1}{a^2}+\frac{1}{c^2}\right),\quad \left(\frac{1}{b^2}+\frac{1}{c^2}\right).$$

5. A ship sails directly north at the rate of (a) miles an hour, and the velocity of the wind is (b) miles an hour; find the direction of the wind so that the vane may point due west.

6. Find the quantity of water discharged from a small given orifice in the side or bottom of a vessel in a given time; the vessel being kept constantly full.

7. Having given the radius of an arc of any colour in the secondary rainbow, find the ratio of the sine of incidence to the sine of refraction when rays of that colour pass out of air into water.

8. If a body revolve in an ellipse (whose major and minor axes are given) with the force tending to its focus, and the time of revolution be given; find the actual velocity of the body at any given point in its orbit.

9. If the hyp. log. $\dfrac{\sqrt{(a^2+x^2)}+a}{\sqrt{(a^2+x^2)}-a}=b$, find x.

10. Find the surface of the solid generated by the revolution of a common cycloid about its axis.

11. Explain why the effect of aberration on a star not situated in the solstitial colure, at six o'clock, either evening or morning, is partly in declination and part in right ascension.

12. A luminous point is placed in the axis of a glass lens, which is *plane* on one side and *curved* on the other; find the nature of the curved surface so that rays diverging from the luminous point, may after passing through the lens, be refracted accurately to another given point.

13. The right ascension and declination of a star being given, as also the time of the year when it rises with the sun; find the latitude of the place.

14. The increment of the hyp. log. (x) =

$$2\left\{\left(\frac{\dot{x}}{2x+\dot{x}}\right)+\frac{1}{3}\left(\frac{\dot{x}}{2x+\dot{x}}\right)^3+\frac{1}{5}\left(\frac{\dot{x}}{2x+\dot{x}}\right)^5+\text{\&c.}\right\}$$

15. Find the following fluents:

$\int\frac{x^2\dot{x}}{(a^2+x^2)^2}$; $\int\frac{a^x\dot{x}}{\sqrt{(1-a^{2x})}}$; $\int z\dot{x}$, where z is an arc whose tangent $=x$.

Monday Evening.—Mr. Fallows.

1. If £1. £8. £27. &c. be lent at the beginning of the first, second, third, &c. year; find the whole

amount due, at simple interest, at the end of n years, at r rate per cent.

2. Given the four sides of a quadrilateral figure inscribed in a circle; to find its diagonals.

3. A string passing over a fixed pulley is coiled, on each side of it, round two cylinders of equal weight (w), the one being of uniform density, the other collected in the circumference; find the tension of the string when they are at liberty to move; the inertia of the string and pulley not being taken into account.

4. Given the area of a right-angled triangle; to find the curve to which the hypothenuse is always a tangent.

5. At what angle must two plane reflectors be inclined, so that a man standing in a given position, may see his face in *profile* in the image of one of them?

6. The ages of two persons being equal; find the value of an annuity of £1. for their joint lives.

7. A body revolves in an ellipse, the force being in the focus; shew that if an additional velocity be communicated to it in its descent from the higher to the lower apse, the apsides are regressive, and if communicated in its ascent from the lower to the higher, they are progressive.

8. Two barometers whose lengths are a, a' inches, contain b, b' inches of air respectively; if on account

of some change in the weather the former barometer falls one inch, what will be the depression in the latter; supposing a perfect barometer to stand at 30 inches before the depression?

9. Equal altitudes of the sun are taken before and after its passage over the meridian, and the times of observation noted by a chronometer; find its error when the change of declination is taken into account.

10. Find the integral of cos. z; and from thence, sum the series cos. a + cos. $(a+b)$ + cos. $(a+2b)$ ...... + cos. $(a+nb)$.

11. Find the following fluents:

$$\int x^2 \dot{x}.\text{arc. (sin.}=x), \quad \int \frac{\dot{x}(1+x^2)}{(1-x^2).\sqrt{(1+x^4)}}.$$

12. Find the relation between x and y, in the following equations:

$$x\dot{y} - y\dot{x} - (x^2+1)\dot{x} = 0;$$

$$(\dot{x}^2+\dot{y}^2)^{\frac{3}{2}} + 2a^{\frac{1}{2}}\sqrt{(a-x)}.\dot{x}\ddot{y} = 0.$$

13. If the mean density of the earth (considered as a sphere) be to the density at the surface as 1 : m; find that power of the distance from its center according to which the density of its parts varies.

14. Explain the construction of *Mercator's* chart, and from thence find the distance of two places projected on the chart whose latitudes and longitudes are given.

15. If a prolate spheroid be cut by a plane passing through the focus, the section will be an ellipse having its focus in the focus of the spheroid.

16. Prove that the sum of the terms of *Taylor's* series commencing with any given term, can always be rendered *less* than the term immediately preceding it.

17. If a small pencil of parallel homogeneal rays be refracted into a sphere of water, and emerge parallel; shew that after two refractions and one reflection, the angle contained between the incident and emergent rays is a *maximum*, and after two refractions and two reflections it is a *minimum*.

18. A circle has the greatest triangle inscribed in it, a circle is inscribed in the triangle, which has the greatest triangle inscribed in it, and so on; find the sum of all the circles and triangles.

19. If $F \propto \frac{1}{D^3}$ and be attractive; shew that six different kinds of orbits may be described with proper velocities and angles of projection, and only six; and when repulsive, only one.

20. A paraboloid rests upon an horizontal plane with its axis vertical and vertex downwards: what must be the length of its axis in order that the equilibrium may be that of *indifference?*

21. If the resistance of the medium vary partly in

the simple and partly in the duplicate ratio of the velocity, and a body urged by the force of gravity ascend or descend in the medium; shew how the spaces described by the body in different times may be compared. *Newton.* Prop. 14. Book II.

22. A rigid prismatic bar of uniform density and given length is placed in the straight line joining two centers of force, whose distance is given, and whose intensities are in the ratio of 2 to 1; find the position of the bar so that it may rest in equilibrium, supposing $F \propto \frac{1}{D}$.

23. The lunar orbit being supposed circular; compare the moon's velocity in quadratures with its velocity at any given place of its orbit, taking into consideration that the earth and moon revolve about their common center of gravity. *Newton.* Prop. 26. Book III.

24. Investigate the following formula for clearing the moon's distance: ver. sin. $(D) =$ ver. sin. $(A-B) + \frac{\cos. A . \cos. B}{\cos. a . \cos. b}$ {ver. sin. (d) — ver. sin. $(a-b)$} where A, B; $a, b,$ are the true and apparent altitudes; D, d the true and apparent distances.

Tuesday Morning.—Mr. Fallows.

FIRST AND SECOND CLASSES.

1. Find two fractions whose denominators are prime to each other and their sum $\frac{32}{45}$.

✓ 2. The area of a trapezium is equal to the product of its two diagonals multiplied by half the sine of the angle formed by their intersection.

3. In the expansion of $\frac{a+bx+cx^2}{1-\alpha x-\beta x^2-\gamma x^3}$; find the general term.

4. Given the lengths of two ordinates of the logarithmic curve and the portion of the abscissa intercepted between them; to construct the curve.

5. Find the *position* of the center of gravity of any number of bodies situated in different planes.

6. If a body fall by the action of an uniform force and describe (a) and (b) feet in the m^{th} and n^{th} second respectively, (reckoning from the beginning of the motion); find the space described in the x^{th} second.

7. Two given glass meniscuses of the same diameter and the same focal length being joined together with their convex sides outwards and the included space being filled with water; find the

focal length of the lens, its thickness not being considered.

8. If an oblate spheroid, whose axes are given, be filled with water and placed with its major axis perpendicular to the horizon; find the time of emptying through a small given orifice at the extremity of the vertical axis.

9. Prove, *strictly*, that $\dot{v}=F\dot{t}$, $v\dot{v}=F\dot{x}$, and $\frac{\ddot{x}}{\dot{t}^2}=F$.

10. In a given latitude and longitude, a vertical plane declines (a^0) from the south towards the west; find the place to whose horizon the plane is parallel.

11. If a body fall from rest through a given space AB towards a given center of force C, in t seconds; compare the force at A with gravity, supposing $F \propto \frac{1}{D^3}$.

12. Investigate the nature of the curve, in which lines drawn from a given point perpendicular to the tangent may always be equal.

13. Find the integral of $\frac{x}{a^x}$.

14. If an elastic chord of uniform density, whose length is (L) and weight (W), be stretched in an horizontal position by a given weight (w) and the increment of length be (l); find the length of the

chord when suspended by one of its extremities; the increment of its length being always as the weight which stretches it.

Tuesday Afternoon.—Mr. Fallows.

FIFTH AND SIXTH CLASSES.

1. A person sold goods to the value of £1000. and gained 20 per cent. What was the prime cost?

2. Surd roots of the form $\pm\sqrt{(b)}$ enter equations by pairs.

3. If two triangles are to each other as their bases; prove that they have the same altitude.

4 A body is projected up a plane inclined to the horizon at an angle of 30^0 with a velocity of 20 feet per second, find where it will be at the end of four seconds.

5. If sin. $(A-B)=\frac{1}{2}$rad. and sin. $(A-B)=$ cos. $(A+B)$, find A and B.

6. Find how far a body will fall from rest, while a pendulum whose length is 20 inches makes 10 vibrations.

7. Define logarithms, and shew from the definition that log. $(ab)=$ log. $a+$ log. b; log. $\left(\frac{a}{b}\right)=$ log. $a-$ log. b; log. $a^n=n$ log. a.

8. A hollow globe is filled with fluid; compare the internal pressure with the weight of the fluid.

9. In the magic lantern, prove that no image will be formed upon the screen, unless the distance between the lantern and the screen be greater than four times the focal length of the lens.

10. The sun is at the same altitude at equal intervals of time before and after its passage over the meridian, supposing no change in declination to have taken place during the interval.

11. If $F \propto \frac{1}{D^2}$; a body revolving in a circle at a given distance from the center will by its motion at any point turned upwards ascend to twice its distance from the center.

12. Find the following fluents:

$$\int \frac{a\dot{x}}{b+\frac{c}{x}}, \quad \int \frac{x\dot{x}}{(1+x^4)^{\frac{3}{2}}}.$$

13. The circumference of a circle to its diameter is nearly in the ratio of 22 to 7.

14. Inscribe the greatest parallelepiped in a sphere.

15. Every inscribed triangle formed by any tangent and the two intercepted parts of the asymptotes of a hyperbola, is equal to a given area.

16. Find the radius of curvature at the vertex of a common parabola.

17. If a body revolve in a logarithmic spiral, find the law of centripetal force tending to the pole of the spiral.

Tuesday Afternoon.—Mr. French.

THIRD AND FOURTH CLASSES.

1. If A, B, C, be the three angles of a plane triangle, having given $\cos. B = \frac{1}{2} \cdot \frac{\sin. A}{\sin. C}$; prove the triangle to be isosceles.

2. Prove that the arc $\frac{60^0}{2^{12}} = \left(2^2 - \frac{1}{2^2}\right)^3$ seconds $= 52''. 44'''. 3^{iv}. 45^{v}$.

3. If a body be projected perpendicularly upward, the time of its ascent through any space is determined from the quadratic equation $(mT^2 - V.T + S = 0)$; shew that the least root is that which answers the conditions of the problem.

4. If a given pendulum be made to oscillate in a cycloid and in a circle, its greatest velocity in the cycloid : its greatest velocity in the circle :: the cycloidal arc described in its descent : the chord of the circular arc described.

5. A solid of revolution, whose axis is perpendicular to the horizon, empties itself by a small given orifice; required its nature, when the velocity of the descending surface varies inversely as the ordinate of the generating figure.

6. An eye being placed so as just to see the lowest point of an hemispherical vessel, when empty; it is required to determine the perpendicular depth of that point of its inner surface nearest to the eye, which is brought into view when the vessel is filled with water.

7. To a spectator in the northern hemisphere, the sun, whose declination = 15^0, rises just two hours before noon; prove that tan. latitude of the place of observation $= \frac{1}{2}\sqrt{3}\sqrt{\frac{(1+\frac{1}{2}\sqrt{3})}{(1-\frac{1}{2}\sqrt{3})}}$. (rad. = 1).

8. A cylinder, whose weight = 133.6 lbs. and rad. = 10, revolves about its horizontal axis; to determine the time in which a weight of 20lbs. acting by means of a string at the circumference of the cylinder, will generate a velocity of 1 foot per second at a distance = 1 from the axis. ($m = 16$ feet.)

9. If bodies move in a logarithmic spiral from different points to its pole, shew that the times of their motion are as the squares of the spaces which they respectively describe.

10. According to what power of the distance must the force vary, that the areas, *dato tempore*, in all

circles uniformly described about the center of force, may be equal?

11. If the point A (Prop. 41. Sect. viii.) be removed to an infinite distance from the center of force, shew from Newton's construction (Cor. 3.), that the hyperbolic spiral will become a circle.

12. The length of the catenary $= a(e^{\frac{D}{a}} - e^{-\frac{D}{a}})$, D being its greatest ordinate and a the lateral tension. Required a proof.

13. $y = \frac{x \cdot \sqrt{(1-x^2)}}{\sqrt{(1-a^2x^2)}}$. Required the *maximum* value of y.

Tuesday Evening.—Mr. French.

1. A person transfers £1000. stock from the *five per cents.* to the *three per cents.* when the former are at 110, and the latter at 84; if, at the end of six months, the *five per cents.* have risen to 112, what must then be the price of the *three per cents.* that he may sell out without having gained or lost by the transfer?

2. Having given two distances from the focus of a parabola and the angle between them; to construct the parabola.

3. To determine the greatest straight line which

can be drawn from a given point in the minor axis of an ellipse to its periphery.

4. A ball is projected from a given point in the horizontal plane at an angle of 30°, and after describing two-thirds of its horizontal range, strikes against a sonorous body; having given the whole interval between the instant of projection, and the instant when the sound reaches the point of projection, to find the initial velocity.

5. The periodic times of four bodies being 24, 22, 20 and 18 days, respectively; in what time, after leaving a conjunction, will they all be again in conjunction, and what number of revolutions will each have performed?

6. If rays fall nearly perpendicularly upon a spherical refracting surface of a denser medium converging to a point between the surface and its center, and sin. I : sin. R :: m : n; shew that the greatest distance between the conjugate foci = $\frac{\sqrt{m}-\sqrt{n}}{\sqrt{m}+\sqrt{n}}.r$: (r being the radius of the refracting surface.)

7. The values of an *oz.* of platina, gold and silver being p, g and s respectively, and their specific gravities a, b, c; compare the value of a coin, made of platina and silver, and which equals a guinea in weight and magnitude, with the value of a guinea.

8. Shew that the n^{th} term in the series of hexa-

gonal numbers is the same with the $(2n-1)^{\text{th}}$ term in the series of triagonal numbers.

9. The point C is such that all straight lines drawn from it to two given points A, B, are in a given ratio; prove that the *locus* of C is the circumference of a circle.

10. A small object is placed at such a point in the diameter of a sphere of water as to be distinctly seen, after one reflection and one refraction, by an eye in that diameter produced; compare its visual angle with the visual angle of the same object when placed in the principal focus of the sphere.

11. Find the following fluents:

$$\int \frac{a\dot{x}}{\sqrt{(1-x^4)}}, \text{ when } x=1; \quad \int \frac{(a^2+x^2)^m \cdot \dot{x}}{(\text{hyp. log. } x)^n}.$$

12. A wheel, in 36 revolutions, passes over 29 yards, and in x of these revolutions it describes $\overset{\text{yds.}}{z} + \overset{\text{feet.}}{y} + \overset{\text{inches.}}{5}$; to find the values of x, y and z.

13. To find the place of a body in an elliptic trajectory at any given time. (*Newton*. Vol. I. Sect. 6.)

14. Deduce *Kepler's* law of the equable description of areas about the center of force from the three fluxional equations of motion.

15. Investigate the expression for the precession in right ascension of a star, whose right ascension is greater than 180° and less than 270°.

16. Required the sum of the terms of a binomial $(a+x)^m$, at intervals of n from each other, beginning with the $(p+1)^{\text{th}}$ term.

17. A given hemispherical vessel, whose thickness is t, resting upon its base, is filled with fluid to a depth = half of its inner radius; required the ratio of the specific gravities of the vessel and the fluid, when the vertical pressure of the fluid = the weight of the vessel.

18. To resolve $(a^2 - ab.2 \cos. \theta + b^2)^{-2s}$ into a series of cosines of arcs, the multiples of θ, by means of the formula $2 \cos. m\theta = x^m + \frac{1}{x^m}$, and the binomial theorem.

19. A body moves in a logarithmic spiral, the centripetal force varying inversely as $(\text{dist.})^2$, and the resistance as the density of the medium and the square of the velocity jointly; from these *data* determine the law of the density.

20. Sum the following series:

$$\frac{8}{1.3.5.7} + \frac{16}{5.7.9.11} + \frac{24}{9.11.13.15} + \&c.$$

to n terms by increments.

$$ax^a + (a+b)\, x^{a+\beta} + (a+2b)\, x^{a+2\beta} + \&c. \text{ to } n \text{ terms.}$$

21. If seven balls be drawn from a bag containing eleven in all, five of which are white and six black; what is the probability that three white balls will be drawn?

22. Prove that the sum of all the numbers of n places, which can be formed with the n digits a, b, c, &c. : sum of all the numbers of n places which can be formed with the n digits p, q, r, &c. of the same scale :: $a+b+c+$&c. : $p+q+r+$&c.

23. In a revolving fluid spheroid of small eccentricity, shew that, if $\sin.^2 \text{lat.} = \frac{1}{3}$, the distance from the center (CP) = the radius of an equi-capacious sphere, and that the central attraction of P arising from the mutual gravitation of the particles of the spheroid, is equal to its attraction to the same sphere at rest.

24. *ABCDE* is a pentagonal billiard-table, with unequal but given sides and angles; it is required to find that point in one of its sides, and the direction of impact, such, that an elastic ball may continually describe the same path, striking every side of the table in succession.

1819.

Monday Morning —Mr. Peacock.

Monday, January 18, 1819.

FIRST AND SECOND CLASSES.

1. What number of degrees, minutes and seconds are contained in an arc equal to radius?

2. If from a point without a parallelogram, lines be drawn to the extremities of two adjacent sides and of the diagonal which they include; of the triangles thus formed, that, whose base is the diagonal, is equal to the sum of the other two.

3. If Mx^{n-m} be the first negative term of the equation

$$x^n + px^{n-1} + \dots - Mx^{n-m} - \dots = 0.$$

and if P be the greatest negative coefficient, then $1 + \sqrt[m]{P}$ is greater than the greatest root of the equation.

4. If the inverse ratio of any two consecutive coefficients of the series

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \&c.$$

be finite, it is always possible to assume x so small, that any one term of the series may exceed the sum of all those which follow it.

5. In the direct collision of bodies, the velocity of the centre of gravity is the same before and after impact.

6. The bulb of a thermometer is successively plunged into boiling water and melting ice, and the mercury in the tube falls a inches: given the diameter of the tube, and the diminution of bulk due to one degree of temperature, to find the capacity of the bulb.

7. If rays nearly parallel, are incident upon a concave spherical reflector, whose radius is r, and if d and d' be the distances of the foci of incident and reflected rays, then

$$\frac{1}{d} + \frac{1}{d'} = \frac{2}{r}.$$

8. Explain what is meant by the line of *collimation*; and shew by what means any error arising from it, may be compensated in the circular transit instrument with an azimuth motion.

9. Explain the method of finding the longitude, by observing the increase of the moon's right ascension, in the interval of her transit over two meridians.

10. Two lines AP and BP in the same vertical plane, pass through two points A and B situated in the same horizontal line: find the locus of the point

P, so that the time of a body's descending down *AP* and ascending up *BP* with the velocity acquired, may be constantly the same.

11. Integrate the differential equation

$$e^x\,dx - \frac{ydy}{e^x} = dy - y\,dx.$$

12. All epicycloids, the radii of whose generating circles bear an assignable numerical ratio to the radii of their bases, are expressible by finite algebraical equations.

13. The cycloid is the curve of quickest descent, between two points which are not in the same vertical line; demonstrate this by the calculus of variations.

Monday Afternoon.—Mr. Peacock.

FIFTH AND SIXTH CLASSES.

1. What is the purchase money of £156. 15*s.* 1*d.* 3 per cent. annuities, at $74\frac{1}{2}$ per cent.?

2. Give the reason why quadratic equations admit of two solutions.

3. Investigate an expression for the number of combinations of *n* things, taken *m* and *m* together.

4. Explain in what case and for what reason,

Cardan's formula for the solution of a cubic equation, does not enable us to determine the roots.

5. Sum the series

$$\frac{1}{\sqrt{2}(1+\sqrt{2})}+\frac{1}{(1+\sqrt{2})(2+\sqrt{2})}+\frac{1}{(2+\sqrt{2})(3+\sqrt{2})}+\text{\&c. in infinitum.}$$

6. Prove that if $2\cos. A=x+\frac{1}{x}$, then $2\cos. mA=x^m+\frac{1}{x^m}$.

7. Explain the method of determining the height of an inaccessible object; give the formulæ of solution of the triangles and adapt them to logarithmic computation.

8. The lines drawn from the angles of a triangle, to the bisections of the opposite sides, all meet in one point.

9. A body descends 400 feet down a plane inclined at an angle of 30^0; calculate the actual time of descent to 3 places of decimals.

10. If W be the weight sustained by the wheels of a carriage, what is the force necessary to keep it at rest, upon a road inclined at a given angle to the horizon, the line of draught being parallel to the road?

11. Explain fully the construction and principle of the common pump.

12. The periodic times of the planetary bodies are independent of the eccentricities of their orbits.

13. Explain the phases of Venus.

14. What is the cause of twilight? Within what limits of polar distance, is there at least one day of the year, when it will continue all night?

15. When parallel rays are incident nearly perpendicularly upon a spherical refracting surface, find the geometrical focus of refracted rays.

16. Investigate the rule for finding the *maxima* and *minima* values of a function of one variable, and shew in what manner they are distinguished from each other.

17. Find an expression for the radius of curvature of the ellipse.

18. Find the centre of gravity of the arc of a cycloid.

19. In the collision of perfectly elastic bodies the relative velocity is the same before and after impact.

20. Given the weight of a body in water and in air, to find its true weight.

21. Compare the forces by which the moon is attracted by the earth and sun.

Monday Afternoon.—Mr. Gwatkin.

THIRD AND FOURTH CLASSES.

1. Extract the square root of

$$\frac{a^2 c}{b} - cf + 2ac\sqrt{-\frac{f}{b}}.$$

2. Solve the equation

$$\frac{\sqrt{(a+bx^n)}-\sqrt{a}}{\sqrt{(bx^n)}}=c;$$

and find x and y from the following

$$\left.\begin{array}{l} x^4-x^2+y^4-y^2=84 \\ x^2+x^2y^2+y^2=49 \end{array}\right\}.$$

3. Produce a given straight line so that the rectangle under the given line, and the whole line produced may equal the square of the part produced.

4. Find by the method of continued fractions a series of fractions converging to $\sqrt{3}$.

5. Prove that the third term of the equation $x^3-px^2+qx-r=0$, cannot be taken away if p^2 be less than $3q$.

6. P and Q sustain each other on two inclined planes, which have a common altitude by means of a string parallel to the planes. Shew from geometrical as well as mechanical considerations that if they be put in motion, their centre of gravity describes a right line parallel to the horizon.

7. Bisect the arc of a semi-cycloid; and if a body oscillate through it, compare the times of describing the first and last half.

8. A right cone whose axis is vertical is just immersed in a fluid, first with its base, then with its vertex downward. Compare the pressure on its whole surface in each case.

9. An object being placed between two plane reflectors inclined at the angle $22^0. 30'$, find the number of images, and shew that two of them coincide.

10. The whole disk of the moon is faintly visible when she is near conjunction, and also when suffering a total eclipse. Explain these phenomena.

11. Find the fluxion of arc whose tang. = $\sqrt{\frac{1-x}{1+x}}$, and shew that $\int dx\,(1-x^2)^{\frac{2n-1}{2}}$ (taken between the limits of $x=0$ and $x=1$) =

$$\frac{1.3.5\ldots.(2n-1)}{2.4.6\ldots.2n}\cdot\frac{\pi}{2}$$

12. Find the area of the curve traced out by the intersection of the sine of an arc, and the secant of half the arc, while the arc increases from 0 to a quadrant.

13. Shew that the number of primes is infinite.

14. Find the polar equation to the ellipse, the centre being considered the pole.

15. Supposing the density of the air to vary as the compressing force, and gravity inversely as $(\text{dist.})^2$ from the earth's center; find the density at any altitude, and shew from the result that the first of the above hypotheses is inadmissible.

Monday Evening.—Mr. GWATKIN.

1. Extract the square root of $14+8\sqrt{3}$.

2. Given the first and last terms, and the sum of an arithmetic series, find the common difference.

3. If three straight lines not in the same plane are equal and parallel, shew that the triangles formed by joining their adjacent extremities are equal and their planes parallel.

4. Shew that the convex surface of a spherical segment is equal to the area of a circle whose radius is the distance from the pole to the circumference of its base.

5. The bodies A, B, C are acted on in parallel directions by the accelerating forces a, b, c; find the point on which, if connected, they would balance.

6. Define a mean solar year, an apparent solar year, an anomalistic year, and a sidereal year. Explain whence arises the difference between the two first, and write down the three last in order of their length.

7. With a single die, find the chance of throwing the six faces in six trials.

8. Given the base of a triangle, and the exterior angle always equal to three times the interior and opposite angle at the base, required the area of the curve which is the locus of the vertex.

9. Find the principal focus of a concavo convex lens of inconsiderable thickness.

10. If a hemispheroid and a paraboloid have the same base and altitude, shew that their solid contents are as 4 : 3.

11. A paraboloid of given dimensions and specific gravity floats with its axis vertical on a fluid whose specific gravity is known. How far may the axis be increased before it tends to fall from its vertical position.

12. If the difference of two numbers be invariable, shew that as those numbers increase the difference of their logarithms diminishes.

13. Integrate the quantities,

$$\frac{dx}{(bx+cx^2)^2}, \quad \cos.^2 x . e^x \, dx, \quad \frac{dx}{\sqrt{a+x-\sqrt{(a^2+x^2)}}};$$

and shew that $\int \frac{dx}{a+b\cos. x} = \frac{1}{\sqrt{(a^2-b^2)}} . \text{arc cos.}$

$\frac{b+a.\cos.x}{a+b.\cos.x}$, a being greater than b.

14. Two planes equal in length are inclined at

45^0 and 30^0 to the horizon. A body is projected downward from the top of the first, and another upward from the bottom of the second, each with the velocity acquired down a vertical line equal in length to either plane. Compare the times of describing each plane, and the velocities at the end of the motion.

15. Shew that Newton's trochoid in the sixth section has a point of contrary flexure, and find its position.

16. Find the length of the meridian for any latitude in Mercator's chart, the oblate figure of the earth being considered.

17. Prove that, in the orbit described round the sun by the centre of gravity of the earth and moon, the elliptical form and the equable description of areas are much more nearly preserved than in that which the earth itself describes.

18. Newton, Sect. 9. Prop. 44. Find the ultimate intersection of Cp the radius vector of the moveable orbit and of the line mn which measures the differential force.

19. Integrate the equations,

$$\sqrt{x}.dy = \sqrt{y}.dx + \sqrt{y}.dy;$$

$$x\frac{dz}{dx} + y\frac{dz}{dy} = n\sqrt{(x^2+y^2)}.$$

20. Define the circle of curvature, and *thence* deduce the expressions for its radius and co-ordinates.

of the centre. Determine whether the circle of curvature cuts the curve at the point of contact or merely touches it; and apply your result to the case of the ellipse at any point and at the extremities of the semi-axes.

21. The earth being supposed spherical and all its matter collected in the surface, in which a circular aperture of given radius is made, and from whose middle point a body being let fall descends to the centre of the earth, find the velocity acquired at any point of the descent.

22. Explain what is meant by the particular solution of a differential equation, and how it arises. Give the method of deducing it, first from the complete integral, and next from the differential equation; and shew that the results thus obtained coincide.

23. Point out the method of determining the max. and min. values of an expression containing two variables; and give the criterion which decides whether the value thus obtained is a maximum, a minimum, or neither.

24. Shew that the planes of the circles which measure the greatest and least curvature of a surface at any point are at right angles to each other; and having given the radii of these, determine the radius of curvature in a plane which is inclined at any angle to the former.

Tuesday Morning.—Mr. Gwatkin.

FIRST AND SECOND CLASSES.

1. Find the price of a marble slab 5ft. 7in. long, and 3ft. 5in. wide, at 6*s*. per square foot.

2. Construct a tetrahedron upon a given straight line, and find the radius of the sphere described about it.

3. A fraction in its lowest terms whose denominator is prime to 10 produces a circulating decimal. Required proof.

4. Find the right line of quickest descent from a right line to a point, the latter line and point being given in position, but not in the same vertical plane.

5. Shew how the focus of a given parabola may be found.

6. Find the weight and magnitude of a solid by weighing it in two fluids whose specific gravities are known.

7. A small rectilineal object is placed before a spherical reflector at a given distance from it and inclined at a given angle to the axis. Required the position and inclination of the image.

8. Given the base of a triangle and ratio of the angles at the base, draw an asymptote to the curve traced out by the vertex.

9. Integrate the following expressions:

$\frac{\sqrt[3]{(1-x^3)}}{x^5} . dx$, $\frac{dx}{\sqrt{(A+Bx+Cx^2)}}$; and solve the equation $x^2 d^2 y = ay dx^2$.

10. Force $\propto \frac{1}{(dist.)^2}$; shew, that if a particle of matter be attracted to a straight line, the direction in which it begins to move is determined by bisecting the angle formed by the lines which join the particle and the extremities of the attracting line.

11. In the expansion of $(1+x+x^2)^n$ write down the coefficient of x^n.

12. Find the centre of gyration of a cube revolving round an axis which passes through its centre of gravity.

13. Sum the series $\tan. A + \frac{1}{2}.\tan. \frac{1}{2} A + \frac{1}{4} \tan. \frac{1}{4} A +$ &c. ad infin.

14. Shew how a plane may be drawn touching the surface of any solid; and draw a plane touching in a given point the surface of an ellipsoid whose equation is $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$; x, y, z, being the co-ordinates, and a, b, c, the semi-axes.

Tuesday Afternoon.—Mr. Gwatkin.

FIFTH AND SIXTH CLASSES.

1. Extract the square root of $x^4 - 2x^3 + \frac{3}{2}x^2 - \frac{1}{2}x + \frac{1}{16}$.

2. Solve the equation,

$$\frac{a}{x} + \frac{\sqrt{(a^2 - x^2)}}{x} = \frac{x}{b},$$

and find the values of x and y in the following equations $x^m y^n = a, \quad x^p y^q = b.$

3. Draw through a given point a straight line making a given angle with a given straight line.

4. A straight line can cut a circle in only two points. Required proof.

5. Trace the changes of algebraic sign, in the sine of an arc, the tangent and secant; and explain why sec. A and sec. $(180^0 + A)$ which coincide should be one positive and the other negative.

6. In the direct impact of a row of perfectly elastic bodies A, B, C, &c. decreasing in magnitude, shew that the momentum communicated to each is less than that communicated to the preceding body. When is the impact of two bodies said to be direct?

7. Shew that the time in which a heavy body descends down the straight line drawn from any point in the surface of a sphere to the lowest point = the time of descent down the vertical axis of the sphere.

8. A straight line is immersed vertically in a fluid. Divide it into three portions that shall be equally pressed.

9. A straight line passes through the principal focus of a spherical reflector at right angles to the axis. Determine the conic section that forms the image. Where must the straight line be placed that its image may be a circle?

10. Given an ellipse, shew how its centre may be found.

11. $y^3 = ax^2 + x^3$. Trace out the curve. Draw an asymptote to it, and find the magnitude and position of the greatest ordinate.

12. Find the fluxion of the log. of $\frac{x}{\sqrt{(1+x^2)}}$, and of an arc whose sine $= 2x\sqrt{(1-x^2)}$.

13. Integrate the following expressions:

$$\frac{x^4\,dx}{x^2+a^2},\quad \frac{x^4\,dx}{(1-x^2)^{\frac{5}{2}}},\quad \text{and}\ \frac{dx}{(x-a)^2.(x-b)}.$$

14. Describe the transit instrument and adjust it to the plane of the meridian.

15. Find the center of gravity of a spherical sector.

16. Two bodies fall to a center of force from the same distance, one acted on by a force varying as the distance, and the other by a force $\propto \frac{1}{(\text{dist.})^2}$. The forces at first being supposed equal, compare the times of descent.

17. Given the velocity, distance, and direction of projection, when the force varies as the distance, shew that the body describes an ellipse; and find the magnitude and position of its semi-axes.

Tuesday Afternoon.—Mr. Peacock.

THIRD AND FOURTH CLASSES.

1. If the roots of the equation

$$x^3 - px + q = 0$$

be real, and if we assume $\cos. \theta = \frac{-q}{2}\sqrt{\frac{27}{p^3}}$, then one of the roots $= 2\cos. \frac{\theta}{3} . \sqrt{\frac{p}{3}}$.

2. Determine the conjugate diameters of an ellipse, which make the least angle with each other.

3. The radius of curvature is a tangent to the evolute.

4. Investigate a general expression for the coordinates of the centre of gravity of the area of a curve, included between a given ordinate and abscissa.

5. Given the quantities and directions of three forces acting upon a material point in different planes, to determine the quantity and direction of the resultant or compound force.

6. In the interior rainbow, the tangent of the angle of incidence is twice that of the angle of refraction.

7. A sphere of less specific gravity than water, ascends from the depth a; what is its velocity at the moment it reaches the surface?

8. Explain the method of determining the heliocentric latitude and longitude of a planet.

9. Enumerate the principal phænomena of Saturn's ring.

10. Find the centre of oscillation of a cylinder of given length and diameter, suspended by its extremity.

11. Prove, that $\tan. nA =$

$$\frac{n \tan. A - \frac{n(n-1)(n-2)}{1.2.3}(\tan. A)^3 + \&c.}{1 - \frac{n(n-1)}{1.2}(\tan. A)^2 + \&c.}$$

12. Find the whole area of the curve whose equation is $a^2 y^2 - a^2 x^2 + x^4 = 0$.

13. Find the locus of the points, in the plane of the moon's orbit, where a body will be equally attracted by the earth and moon.

Tuesday Evening.—Mr. Peacock.

1. If two spherical triangles have two sides of one triangle equal to two sides of the other, each to each, and the included angles equal, the triangles are equal in every respect.

2. The modulus of tabular logarithms or

$$M = .4342944819;$$

shew in what manner this number is determined.

3. It is always possible to find those roots of numerical equations, which are whole numbers or rational fractions, without the aid of formulæ of approximation.

4. Explain the method of determining the position of the nodes of the moon's orbit: what is the physical cause of their retrograde motion?

5. The friction of a body being supposed independent of velocity, to find an expression for the time of a body's descent down a given inclined plane, the friction being equal to $\frac{1}{n}$th part of the pressure.

6. A cubical iceberg is 100 feet above the level of the sea, its sides being vertical: given the specific gravity of sea water $= 1.0263$ and of ice $= .9214$, at the temperature of 32^0, to find its dimensions. Is this position one of stable equilibrium?

7. Prove that the centres of oscillation and suspension are reciprocal. Of what use is this property, in the determination of the length of a pendulum which vibrates seconds in any given latitude?

8. Explain the method of determining the ratio of the sines of incidence and refraction both in liquid and solid bodies.

9. Given the latitudes and longitudes of two places, where the inclination of the magnetic needle is nothing, to find the point of the terrestrial equator, which is cut by the magnetic equator, supposing it a great circle of the earth.

10. Of all equal quadrilateral figures, the square has the least perimeter.

11. Integrate

(1.) $\frac{dx}{x\sqrt{(bx^2-a)}}$, and $\frac{d\theta}{(\sin.\ \theta)^4 \cos.\ \theta}$.

(2.) $\frac{dx}{\sqrt{(a^4-x^4)}}$ from $x=0$, to $=a$.

(3.) $\frac{\left(1+\frac{dy^2}{dx^2}\right)^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{a^2}{2x}$.

(4.) $\frac{dx}{\sqrt{(1-x^2)}} + \frac{dy}{\sqrt{(1-y^2)}} = 0$.

(5.) $xy - \frac{d^2z}{dx\,dy} = 0$.

12. Find the equation of the curve which is the locus of the extremities of the perpendiculars from the centre upon the tangents of the equilateral hyperbola, and determine the position of its tangents at the points where it cuts the axis.

13. Given

$$\log. 510 = 2.70757018$$
$$\log. 511 = 2.70842090$$
$$\log. 513 = 2.71011737$$
$$\log. 514 = 2.71096312$$

to find the logarithm of 512, by the method of interpolations.

14. Explain the principle and construction of the achromatic telescope.

15. What is the least velocity with which a body must be projected from the moon, in the direction of a line joining the centres of the earth and moon, so that it may reach the earth?

16. If the bulb of a thermometer be a sphere, whose diameter is 1 inch, and if the diameter of the tube be $\frac{1}{10}^{\text{th}}$ of an inch, what is the pressure upon the interior of the bulb, when the mercury stands at the altitude of 10 inches above it, exclusive of that portion of the pressure which sustains the mercury in the tube?

17. If $nt = u + e \sin. u$, where u is the eccentric and nt the mean anomaly, apply *Lagrange's* Theorem to the developement of $a(1 - e \cos. u)$, in terms of cosines of nt and its multiples.

18. Prove, that in going from the equator to the pole, the increment of gravity varies very nearly as the square of the sine of the latitude. In what manner does this variation affect, 1st, the length of a pendulum vibrating seconds, and 2dly, the altitude of the barometrical column ?

19. Prove, that there can be no more than five regular solids; and find the angles which their terminating planes make with each other.

20. Given the weight of the key-stone of a circular arch, in a state of perfect equilibration, and the angles formed by each of its faces with a vertical line; to find the horizontal pressure upon the abutments.

21. Prove, that

$$\tan.^{-1}\frac{x}{y} = \tan.^{-1}\frac{ex-y}{ey+x} + \tan.^{-1}\frac{e_1-e}{ee_1+1} + \tan.^{-1}\frac{e_2-e_1}{e_1e_2+1} + \ldots\ldots\ldots + \tan.^{-1}\frac{e_n-e_{n-1}}{e_{n-1}e_n+1} + \tan.^{-1}\frac{1}{e_n};$$

where $\tan.^{-1}\frac{x}{y}$ represents an arc whose tangent is $\frac{x}{y}$, and where $e, e_1, e_2, \ldots e_n$ are any numbers whatever.

22. A spherical shell with a small orifice at it's lowest point, is filled with air of the density of the atmosphere, and immersed in water to a depth a: with what velocity will the water rush into the shell, and what portion of the sphere will it occupy, when the motion ceases?

23. Develope $\frac{x}{e^x - 1}$ in a series involving ascending powers of x. Of what use are the coefficients of this series in expressing the law of the coefficients of the series for tan. θ in terms of θ?

24. Enùmerate, as Newton has done, the principal proofs of the truth of the theory of universal gravitation.

1820.

Monday Morning.—Mr. Whewell.

Monday, January 17, 1820.

FIRST AND SECOND CLASSES.

1. Given two sides and the included angle, find an expression for the area, (1) in a plane, and (2) in a spherical triangle.

2. A straight line cuts a parabola, whose vertex is A, in two points P and Q, and its axis in O; ordinates PM, QN, being drawn, shew that AO is a mean proportional between AM and AN.

3. The force varies inversely as $(\text{distance})^{\frac{10}{3}}$. A body is projected from any point in any direction, with a velocity equal to that from infinity. Find the position of the apse, and the whole angle described.

4. On a horizontal dial the angle corresponding to a second of time at 4 o'clock, is double the angle for a second at noon. Find the latitude of the place.

5. The equation to a curve is $y=x\sqrt{\frac{a^2-x^2}{a^2+x^2}}$. Trace it; find its maximum ordinate, and its area.

6. The earth revolving round a fixed axis, shew that a body let fall from the top of a high tower will not strike the ground exactly at the foot of the tower. Between what cardinal points of the compass will the point struck be situated with respect to the foot of the tower?

7. Express the distance of a point from the earth's center in terms of the latitude.

8. A point T moves uniformly along a straight line; another point P, with *three* times the velocity, always moves towards T, so as to describe the *curve of pursuit*. Trace the curve, and shew that the path described by T from the time when the paths are at right angles till it is overtaken by P is $\frac{3}{8}$ of their distance at that time.

9. The equation to the elliptical paraboloid being $ax^2+by^2+abz=abc$, draw a normal to it; and determine the points where this line cuts the three co-ordinate planes. Also find the solid content of a portion contained by planes parallel to the planes of xz and yz.

10. Find right-angled triangles, such that all the sides shall be rational numbers.

11. If a, b, c be the sides of a plane triangle and

C the angle opposite to c; prove that

$$\text{hyp. log. } c = \text{hyp. log. } a - \frac{b}{a}\cos. C - \frac{b^2}{2a^2}\cos. 2C - \frac{b^3}{3a^3}\cos. 3C - \&c.$$

12. Integrate the differential

$$dx\sqrt{\frac{1-e^2x^2}{1-x^2}},$$

in a series which converges rapidly when e is nearly $=1$; and the equation

$$(a+y)\frac{dx}{dy} = x + y - x\frac{dy}{dx}.$$

13. Define the moon's *variation.* Give Newton's construction for it, and hence shew how it varies.

Monday Afternoon.—Mr. Whewell.

FIFTH AND SIXTH CLASSES.

1. Find the value of .151636363, &c. of £1.

2. Find in what time at compound interest, at 5 per cent. a sum will become 10 times its original value. (N. B. the log. of 105 is 2.0211893.)

3. Solve the equations

$$x + \sqrt{\{x^2 + \sqrt{(x^2+96)}\}} = 11$$

$$\left.\begin{array}{l} x(y+z)=a \\ y(x+z)=b \\ z(x+y)=c \end{array}\right\}$$

$x^3 - 6x - 40 = 0$, by Cardan's method.

$3^{2x} . 2^{3x} = 10.$

4. A beam rests with one end on a horizontal plane, and the other against a vertical wall; find the horizontal force necessary to prevent its lower end from sliding outwards.

5. A projectile is to be thrown across a plain 120 feet wide, to strike a mark 30 feet high, the velocity of projection being that acquired down 80 feet; find at what angle it must be projected.

6. A piece of wood weighs 12 lbs. and when annexed to 22 lbs. of lead, and immersed in water, the whole weighs 8 lbs. The specific gravity of lead being 11 times that of water, find the specific gravity of the wood.

7. A cylinder whose axis is horizontal empties itself by a hole in the lowest part; find the time.

8. A trapezium has two opposite sides equal, and the other two parallel; compare the resistance upon it, when it moves in the direction of the parallel sides, and when it moves in a direction perpendicular to them.

9. Explain why all parts of the field of view of a telescope are not equally bright; and find the proportion of the bright part to the whole in the astronomical telescope.

10. Having observed the elongation of a planet when stationary, shew how its distance from the sun may be found.

11. Find the integrals of

$$\frac{dx}{\sqrt{a}-\sqrt{x}}, \quad \frac{x\,dx}{\sqrt{(2ax-x^2)}}, \quad dx.\log.x.$$

12. Find an expression to determine the force by which a body may be retained in a given curve: apply it to the curve whose equation is

$$p^2 = \frac{b^2 x^4}{a^4+x^4}.$$

13. The force varying inversely as the distance, find the angle between the apsides in an orbit nearly circular, and prove the method.

14. Sum the following series to n terms,

$$\frac{1}{\sqrt{x}} - \frac{\sqrt{2}}{x} + \frac{2}{x\sqrt{x}} - \&c.$$

and continue the harmonic progression.... 3, 4, 6, ... upwards and downwards. How far can it be continued either way?

15. Form the equation, whose roots are

$$2+\sqrt{-3},\ 2-\sqrt{-3},\ 1, \text{ and } -5.$$

16. In any plane triangle of which the sides are a, b, c, and the opposite angles A, B, C, prove that

$$\sin. A = \frac{a.\sin. C}{\sqrt{(a^2-2ab.\cos. C+b^2)}}.$$

17. How high will a given balloon ascend? When it floats in the air, supposing that a given weight of ballast is thrown out, to what additional height will it rise, and how much will a barometer in it sink?

Monday Afternoon.—Mr. Wilkinson.

THIRD AND FOURTH CLASSES.

1. Find the discount of £.100 for one year at 5 *per cent.* and then calculate the interest on this discount for the same time.

2. Solve the equations

$$\frac{x^4+1}{(x+1)^4} = \frac{1}{2}, \text{ and } \sqrt{(x^4-1)}+\sqrt{(x^2-1)}=x^3.$$

3. If a rigid sphere, revolving round an axis, become fluid and therefore change its figure, the whole moment of inertia will remain the same as before.

4. Find the centers of oscillation of a square suspended by one corner and oscillating *flatways* and *edgeways*.

5. What is the reason that waves always break against the shore whatever be the direction of the wind?

6. The horizon of any place being taken as the plane of projection, find the figure and dimensions of the path of the diurnal motion of a given star orthographically projected.

7. What is the meaning of the astronomical term equation? The equation of time (arising from what causes?) is a maximum about the beginning of November, is it then additive or subtractive?

8. The number of impossible roots of any equation $x^n - px^{n-1} + \&c. = 0$ is not increased by multiplying its terms by the successive terms of the series 0, 1, 2, 3, 4, &c.

9. Integrate these expressions

$$\frac{dz}{1 - \tan.^2 z}, \quad \text{and} \quad \frac{dx}{1 + x + x^2}.$$

10. If a tangent be drawn at the extremity of the latus rectum of a conic section meeting the tangent at the vertex, the part of this latter tangent thus cut off shall be equal to the distance between the vertex and the focus of the curve.

11. Find in what curve a body must revolve round a repulsive force varying as the distance from a point, so that its velocity may always equal that in a circle at the same distance, round an equal attractive center of force.

12. Transform the equation to the lemniscata $(x^2 + y^2)^2 = a^2 . (x^2 - y^2)$, from rectangular to polar co-ordinates.

13. If in an equation between x and y the sum of the indices be the same in every term, the loci of the corresponding values of x and y are straight lines.—Find what lines are defined by the equation $y^3 - 2xy^2 + x^3 = 0$.

Monday Evening.—Mr. WILKINSON.

1. What is the purchase of £1034. 15*s*. stock in the 3 per cents. at $62\frac{1}{2}$?

2. If two straight lines in space be parallel, their projections on any plane will be parallel.

3. Find the solidity of an octahedron.

4. Required the present worth of an annuity of (a) pounds for n years payable every instant in equal portions, interest also being convertible into principal as fast as it becomes due.

5. R and r being the radii of the circumscribed and inscribed circles of the triangle whose sides are a, b, c, shew that

$$Rr = \frac{abc}{2(a+b+c)}.$$

6. If n be a prime number and a and b integers not divisible by n, then $\dfrac{a^{n-1} - b^{n-1}}{n}$ is a whole number.

7. State D'Alembert's principle, and the principle of virtual velocities, and employ them in deducing this theorem $4m(Ax + By + Cz) = Aa^2 + Bb^2 + Cc^2$; A, B, C, being *weights* which put a system in motion, x, y, z, the spaces perpendicularly descended by them respectively, and a, b, c, their actual velocities.

8. A slender rod of uniform thickness revolves round an axis passing through one of its extremities; find,

(1.) At what point an obstacle must be opposed to it that there may be no stress on the axis from the shock?

(2.) What quantity of matter should be collected in this point that the impulse on the obstacle may be the same as that of the rod?

(3.) At what distance from the axis the obstacle must be opposed that the impulse may be the same as if the whole matter in the rod were collected in that point?

9. The earth being supposed a sphere of uniform density, shew that the pressure on each side of a plane passing through its center : the whole weight of the earth :: 3 : 16.

10. A piston is thrust down uniformly into a cylinder full of air, having a small orifice at the end; find the quantity discharged in a given time into a vacuum.

11. If parallel rays fall upon a spherical refracting surface, the distance from the axis of the geometrical focus of a small pencil which does not pass through the center, is proportional to the cube of the distance at which it is incident from the axis.

12. The sun's right ascension on two successive days at noon was $6^h. 40'. 25''$, and $6^h. 45'. 13''$, by

the Nautical Almanack (and therefore in sidereal time); the moon's right ascension at the same time was from the Nautical Almanack (and therefore expressed in degrees) $5^s.\ 9^0.\ 32'$, and $5^s.\ 20^0.\ 9'$. Required the time of the moon's transit in the interval.

13. The length of a degree perpendicular to the meridian is always greater than the degree of the meridian corresponding.

14. Define the axis of a curve, and draw the axis to the curve of which $y^3 - 3axy + x^3 = 0$ is an equation.

15. T and t are the parts of the tangent at the vertex (A) of a rectangular hyperbola, (whose semi-axis $= 1$) cut off by lines (CP, Cp) drawn from the center to the curve; shew that if the sector CAP be n times the sector CAp,

$$\frac{1-T}{1+T} = \left(\frac{1-t}{1+t}\right)^n.$$

16. A's skill is double B's, and their stakes equal; find what C, whose skill is equal to A's, must stake, that A's advantage may be as great as if he played with B.

17. If it be an even wager that D wins (n) successive games of E, what is E's chance of winning the first game?

18. Integrate the differential equations

$$\frac{dy}{dx} = a \sin. x + by; \text{ and } \frac{d^2 y}{d x^2} - \frac{2dy}{dx} + 2y = 0;$$

and shew how to separate the variable quantities in

$$\frac{dy}{dx} = \frac{a^2 + x^2 - y^2}{a^2}.$$

19. Apply the method of increments to sum the series

$\frac{3}{1.2} \cdot \frac{1}{2} \cdot \frac{4}{2.3} + \frac{1}{2^2} + \frac{5}{3.4} \cdot \frac{1}{2^3}$ + &c. (to n terms), and also to shew that sin. x + sin. $3x$ + sin. $5x$ + ... (to n terms) $= \frac{\sin.^2 nx}{\sin. x}$.

20. If tangents be drawn at the extremities of the major axis of an ellipse, the rectangle under the parts of these tangents intercepted by the tangent at any other point of the curve is a constant quantity.

21. Conversely, if AT, Vt, be two perpendiculars to a given line AV and on the same side of it, and also the rectangle $AT \times Vt$ be constant; to investigate the nature of the curve which the line Tt perpetually touches.

22. The axes of two equal cylinders intersect each other at right angles, find the content of the solid cut out of each by the surface of the other.

23. The attractions of *ellipsoids* on particles in the surface in directions perpendicular to any of the principal sections are as the distances.

24. Two bodies connected by an inflexible rod

without weight, and to one of which a certain velocity is communicated, are constrained to move along two grooves at right angles to each other; required the circumstances of motion; and shew that when the bodies are equal the line which joins them revolves uniformly.

Tuesday Morning.—Mr. Wilkinson.

FIRST AND SECOND CLASSES.

1. What will the ceiling of a room come to, whose length is 24ft. 6 in. and width 16ft. 3in. at 3*s*. per yard?

2. $u = \text{arc}\left(\tan. \frac{x-y}{x+y}\right)$ find the differential of u.

3. Every positive number has an infinite number of logarithms, only one of which is real, and every negative number has only imaginary logarithms.

4. When a system of bodies connected in any manner is in equilibrium the center of gravity is as high or as low as is possible.

5. The resistance on a cube moving in a fluid in the direction of its diagonal : resistance on the same cube moving in a direction perpendicular to its side :: 1 : $\sqrt{3}$.

6. A conical vessel is filled with water; find that

heavy sphere which when put into it shall force out the greatest quantity of fluid.

7. Draw a tangent to the curve formed by the intersection of a right cone with the cylinder erected on the radius of the base as a diameter.

8. Investigate the nature of the curve trisecting all the arcs described on the same chord.

9. Integrate

$$\frac{ydx - xdy}{(x+y)^2}, \quad \text{and} \quad xdy - ydx = \frac{2xdy - ydx}{\sqrt{(x^2+y^2)}}.$$

10. A, and B whose skill is m times A's, agree to play with this law, that A shall continue to stake against B so long as B wins without interruption; shew that B's expectation is worth $(m-1)$ times the stake.

11. A circle of curvature is described at the vertex of a parabola, and another circle which touches that and both the arcs of the curve, and so on continually; compare the sum of all the areas of these circles with that of the parabola.

12. If D and D' be the lengths of a degree of a meridian at the equator and in latitude λ respectively, a and b the equatorial and polar diameters,

$$\frac{a}{b} = \frac{\sin.\lambda}{\sqrt{\left(\frac{D}{D'}\right)^{\frac{3}{2}} - \cos.^2\lambda}}.$$

13. Explain how the comparative densities of the

sun and moon have been deduced from the phænomena of precession and nutation.

Tuesday Afternoon.—Mr. Wilkinson.

FIFTH AND SIXTH CLASSES.

1. Required the value of 1cwt. 2qrs. 3lbs. at £4. 5*s.* 6*d.* per cwt.

2. Find the value of

$$\frac{x - x^5}{1 - x^2} \text{ when } x = 1.$$

3. Write down the 4 first terms of the expansion of

$$\left(x - \frac{1}{x}\right)^{-\frac{1}{2}}.$$

4. The two bases of any oblique prism are reciprocally proportional to the sines of the angles which they make with the axis.

5. Given the forces of many agents, to find the time in which they will all produce a given effect.

6. Find the time of one oscillation in a cycloid.

7. A row of non-elastic balls whose magnitudes increase in geometrical progression are placed at equal distances in a straight line, and a given velocity is communicated to the first; required the time before the n^{th} is put in motion.

8. Explain the method of finding the specific gravity of a body lighter than water.

9. Construct Newton's telescope, and shew that objects appear inverted through it.

10. The aberration of a given star in right ascension is not necessarily nothing when that in declination is a maximum.

11. Integrate

$$\frac{dx}{x^2+1}, \quad \frac{dx}{\sqrt{(x^2+1)}}, \quad \text{and } \frac{dx}{x\sqrt{(x^2+1)}}.$$

12. Find the area and point of contrary flexure of the curve, whose abscissa is always equal to the arc of a circle, the versed sine of which is the ordinate.

13. If $S = 1 + \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} +$ &c. *in inf.*

and $s = 1 - \frac{1}{2^m} + \frac{1}{3^m} - \frac{1}{4^m} +$ &c. *in inf.*

Shew that $S : s :: 2^{m-1} : 2^{m-1} - 1$.

14. The total number of odd combinations that can be formed out of any number of things is greater by unity than the total number of even.

15. If at any point P in an ellipse the ordinate NP be produced to meet the tangent at the extremity of the latus rectum, the whole line thus produced is equal to SP the focal distance.

16. The force to a plane must vary inversely as the cube of the distance in order that the trajectory may be a semi-circle.

17. If two bodies describe about each other and about their common center of gravity similar and concentric ellipses, the forces with which they attract each other are proportional to their distances.

Tuesday Afternoon.—Mr. Whewell.

THIRD AND FOURTH CLASSES.

1. In the common system of notation explain why the number of digits cannot be more or less than the local value 10.

2. To find a point within a given triangle from which the three sides shall subtend equal angles.

3. A body is projected with the velocity due to a height h, at an angle A with the horizon. Find an expression for the latus rectum of the parabola described.

4. Shew how the points of contrary flexure in spirals may be found; and apply the method to find the point of flexure of the *lituus* whose equation is $\theta = \frac{a^2}{r^2}$, θ being the angle, and r the radius vector.

5. Shew that a body cannot move so that the

velocity shall vary as the space from the beginning of motion. And if the velocity vary as the cube root of the space, find how the time and the force vary.

6. Two bodies *P* and *Q*, whose specific gravities are *m* and *n*, balance each other on a given straight lever. When the whole is immersed in water whose specific gravity is 1, what alteration must be made in the place of the fulcrum, that they may continue to balance?

7. If a right cone, the diameter of whose base is *BC*, and vertex *A*, be cut by a plane so that the section may be an ellipse whose major axis is *PQ*, the solid content of the part towards the vertex is to that of the whole, as $(APQ)^{\frac{3}{2}}$ to $(ABC)^{\frac{3}{2}}$; *APQ* and *ABC* being the areas of those triangles. Also find the equation to the surface of a cone referred to three rectangular co-ordinates.

8. A beam hangs, by means of a given cord fastened to its upper end, from a fixed point in a vertical wall. Against what point of the wall must its lower end be placed, that it may have no tendency either to ascend or descend? Within what limits for the length of the beam is this equilibrium possible?

9. The equation $x^4 - 45x^2 - 40x + 84 = 0$, has two roots whose difference is 3; find them.

10. The force varies inversely as the square of the distance. A body is projected in a direction which

makes an angle of 60^0 with the distance, with a velocity which is to the velocity from infinity as $1 : \sqrt{3}$. Find the major axis, the position of the apse, and the excentricity, of the ellipse which will be described; and the periodic time.

11. Integrate

$$\frac{dx}{(1-x^3)^{\frac{1}{3}}}; \quad e^{\sqrt{x}} x dx; \quad \sin. mx. \sin. nx. \cos. px. dx;$$

$$\frac{dy}{dx} = \frac{a + 2x - y}{a - x + 2y}.$$

12. Sum the series

$$\frac{1}{1.3} - \frac{1}{2.4} + \frac{1}{3.5} - \&c. \textit{ in inf.}$$

$$\frac{2}{1.3.7} + \frac{4}{3.7.15} + \frac{8}{7.15.31} + \&c. \text{ to } n \text{ terms}$$

by increments:

and prove $1^2 + n^2 + \left(\frac{n(n-1)}{1.2}\right)^2 +$

$$\left(\frac{n(n-1).(n-2)}{1.2.3}\right)^2 + \&c. = \frac{1.2.3\ldots 2n}{(1.2.3\ldots n)^2}.$$

13. If we divide a, a^2, a^3,.... by a prime number p, (a being any number) we shall obtain a remainder 1 before we have taken p terms. Also after this remainder the remainders will recur.

Tuesday Evening.—Mr. Whewell.

1. The area of any right angled triangle is equal to the rectangle of the semi-perimeter and the excess of the semi-perimeter above the hypotenuse. Required proof.

2. *A* sets off from London to York, and *B* at the same time from York to London: they travel uniformly; *A* reaches York 16 hours, and *B* London 36 hours, after they have met on the road; find in what time each has performed the journey.

3. The surface of a right cone being given, to find its form that the solid content may be the greatest possible.

4. The equation to a straight line being $y=ax+b$, find the equation to another straight line drawn perpendicular to the first and passing through a given point. Also, solve the same problem when the lines are referred to *three* rectangular co-ordinates.

5. Two given spheres are moving in given straight lines with given uniform velocities; find where they will meet, (1) when their directions are in the same plane, (2) when they are not.

6. A slender cylinder, whose specific gravity is $\frac{1}{2}$ that of water, is placed in the fluid in an oblique position: find the magnitude, direction, and point

of application of the force which must act on the cylinder to keep it immersed $\frac{5}{6}$ of its length.

7. A gate of given weight and form is hung by hinges to a post inclined at a given angle from the vertical. When it swings freely, find the time of its small oscillations.

8. The perihelion distance of a comet is $\frac{1}{3}$ the distance of the earth from the sun; and its orbit, which is parabolical, and the earth's, which is circular, are in the same plane: how many days is the comet within the earth's orbit?

9. Describe the *repeating circle*, and the method of observing with it; and explain its advantages.

10. Find the time of a body falling through any space towards a center of force, (1) when the force varies inversely as the square root, (2) when it varies inversely as the cube root, of the distance. For what laws of force is the integration, which gives the time, practicable?

11. The force varies inversely as the fifth power of the distance. A body is projected with a velocity which is to the velocity from infinity as 5 to 3, and in a direction which makes with the line drawn to the center an angle whose sine is $\frac{2\sqrt{6}}{5}$. Find the orbit, and the time of describing a given angle; and determine whether it has an apse.

12. AM, MP, MT, are the abscissa, ordinate,

and subtangent of a curve, of which the property is that $AM : MP :: MT : TA$; find its equation, and trace it.

13. Integrate the following differentials and differential equations,

$$\frac{dx\sqrt{(1-x^2)}}{1+x^2}; \ \epsilon^{ax} \sin.^2 x . dx;$$

$$x^m(ydx+xdy)=y^n(ydx-xdy);$$

$$d^3y - 3\,d^2y\,dx + 3\,dy\,dx^2 - y\,dx^3 = 0;$$

$$x\frac{dz}{dx}+y\frac{dz}{dy}=z.$$

14. If P and S, attracting each other with forces proportional to their masses, revolve round their center of gravity; their periodic time is to the periodic time of an indefinitely small particle, which describes a similar orbit round S at the same distance, as $\sqrt{S}$ to $\sqrt{(S+P)}$. Also, if to P be annexed an equal mass which is *not attracted* by S, what alteration will be made in this proposition?

15. In any polyhedron the number of solid angles together with the number of plane faces, exceed by 2 the number of *edges*.

16. On two straight lines at right angles to each other, two points move respectively from given positions, with equal uniform velocities; find the curve to which the line which joins them is always a tangent.

17. In a bag are 8 bank notes, viz. 1 of twenty pounds, 2 of five, and 5 of one: a person is allowed

to take out three indiscriminately: what is the value of his expectation?

18. The radiating point and the caustic being given, shew that there are an infinite number of reflecting curves which will produce the caustic.

19. A body falls towards a center of force which varies directly as the distance, in a medium of which the resistance varies as the square of the velocity: find the velocity at any point. Shew from your result that when the resistance vanishes, the velocity coincides with that in a non-resisting medium.

20. Let $yz = \phi\left(\frac{y}{x}\right)$ be the equation to a curve surface, where ϕ is any function whatever: shew that the part which the tangent plane to any point cuts off from the axis of z, is twice the value of z for that point.

21. Sum the series,

$$\frac{1}{1.4} - \frac{1}{3.6} + \frac{1}{5.8} - \text{\&c. } in\ inf.$$

$$x.\cos. A + x^2.\cos. 2A + x^3.\cos. 3A + \text{\&c. } in\ inf.$$

Also, prove that

$$\frac{1}{1^2+1} + \frac{2}{2^2+1} + \frac{3}{3^3+1} + \text{\&c. } in\ inf.$$

$$= \frac{\pi}{2} \cdot \frac{\epsilon^{2\pi}+1}{\epsilon^{2\pi}-1} - \frac{1}{2};$$

and find an expression for the n^{th} term of a series where each term is the product of the two preceding.

22. AB is the axis of a cycloid, (of which A is the vertex,) and C its middle point. An ordinate is drawn meeting the axis in M, the cycloid in P, P', and the generating circle in Q, Q'. CN is taken towards A equal to AM. Then the cycloidal sector PNP' = triangle QBQ'. Required proof.

23. If a chain of given length be suspended from two points, shew that its center of gravity is lowest when its form is a catenary.

24. Expand the radius vector of an ellipse about the focus $\left(=r=\frac{a(1-e^2)}{1+e.\cos.\theta}\right)$ in a series of the form

$$A+B.\cos.\theta+C.\cos.2\theta+D.\cos.3\theta+\&c.$$

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